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SIMULATION STUDY OF A HYDRO-ELECTRIC SYSTEM

by



E. T. Quinlan

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES
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FACULTY OF GRADUATE STUDIES

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled Simulation Study of a Hydro-Electric System submitted by E. Quinlan in partial fulfillment of the requirements for the degree of Master of Science.



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Abstract

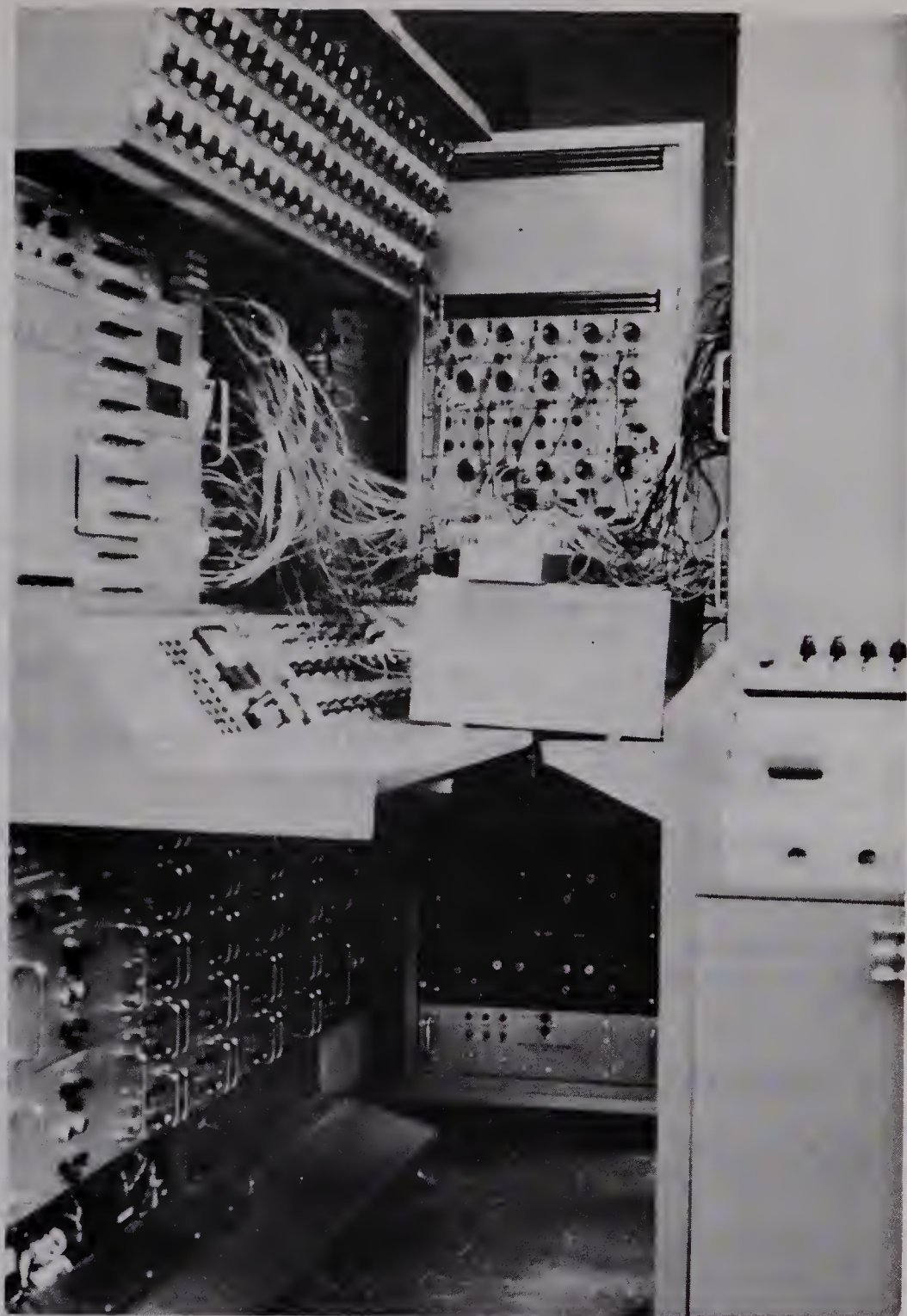
Electronic analogue computers and additional circuitry are employed to simulate two interconnected hydro-power plants acting under the influence of a load-frequency time controller. The controller which was developed by Calgary Power Co. Ltd. is unique, to the author's knowledge. The units in the individual plants are assumed to act as one due to synchronizing power between machines. It was further assumed that the generators of the two plants have identical characteristics.

A study was made to obtain controller settings for optimum plant operation for different hydro and governor characteristics.

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GENERAL VIEW OF EQUIPMENT

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CHAPTER I

Section I -I Introduction

The two major aspects of power system regulation are frequency and load control. Due to the advent and popularity of synchronous clocks, it has become of increasing importance to supply correct time to the consumer. The system frequency is adjusted by raising or lowering the permanent speed droop curve as required. The load apportionment between units is originally set by the slope of the above curve, however, it may be altered by applying a bias signal to frequency control function. Correct time can also be incorporated by temporary over-regulation of the frequency.

The frequency on modern power systems is held within 0.1 cycles per second. This exceeds consumer requirements in regard to frequency. As already indicated, this in itself does not provide for accuracy of synchronous clocks as their time is the integral of frequency.

The control unit devised by Calgary Power Limited is capable of providing frequency tolerances considerably better than 0.1 cycles per second, load sharing between units, and correct time to within 5/8 of a second, based on the assumption that rapid system load changes do not occur. Synchronous time rather than frequency is the basis of operation. If a time error exists, the governor's permanent speed droop is adjusted to correct for the deviation in frequency and a second signal is applied to cause the unit to overspeed to correct the time error.

Load control is provided by applying a bias signal to the controller such that every generator may operate on a different time datum.

It is the purpose of this analysis to determine optimum controller settings for system operation, with respect to frequency, time, load distribution and power disturbances. To accomplish this it was necessary to simulate a system. Therefore a method of simulation was first decided upon, followed by the development or acceptance of the required building blocks: generator, voltage regulator, hydro-system, load and controller. The system was then tested by applying consecutive load changes of 25% and 15%.

To facilitate the interpretation of the results, the controller's action on the governor is converted to equivalent frequency error signals. To generalize the results, all simulated quantities except time and synchronizing torque angle are expressed in per unit quantities. However, in the development of the power system building blocks, it is sometimes necessary to represent the quantities in their respective units. Thus to avoid confusion in the forthcoming analysis, all quantities will be assumed to be expressed in their units unless it is specifically indicated that they are expressed in per unit.

The following tests were made on the system.

Test	Unit #1						Unit #2					
	Sp	k	V	γ	T _w	m	Sp	k	V	γ	T _w	m
1	4	2	95	100	0.60	10	4	2	95	100	2.00	10
(2)	4	2	95	100	0.60	10	4	2	95	100	0.60	10
3	4	2	95	100	0.60	10	4	2	95	100	1.33	10
4	4	2	95	100	0.60	10	4	2	95	100	1.00	10
(5)	4	4	95	100	0.60	10	2	4	95	100	0.60	10
6	4	2	95	100	0.60	10	2	4	95	100	0.60	10
7	4	2	95	100	0.60	10	2	4	95	100	1.00	10
(8)	4	2	95	100	0.60	10	2	4	95	100	2.00	10
9	4	2	95	100	0.60	10	4	2	95	60	0.60	10
10	4	2	95	100	0.60	10	2	4	95	60	0.60	10
11	4	2	95	100	0.60	10	2	4	95	60	2.00	10
(12)	2	4	95	100	0.60	10	4	2	95	60	2.00	10
(13)	2	4	95	60	0.60	10	4	4	95	100	2.00	10
14	4	2	285	100	0.60	10	4	2	285	100	0.60	10
(15)	4	4	47	100	0.60	10	4	4	47	100	0.60	10
(16)	4	4	95	100	0.60	10	4	4	95	100	2.00	10
17	4	6	95	100	0.60	10	4	6	95	100	0.60	10
18	4	8	95	100	0.60	10	4	8	95	100	0.60	10
19	4	8	143	100	0.60	10	4	8	143	100	0.60	10
20	4	8	120	100	0.60	10	4	8	120	100	0.60	10
(21)	4	10	95	100	0.60	10	4	10	95	100	0.60	10

Test	δp	k	V	γ	T_w	m	δp	k	V	γ	T_w	m
(22)	4	2	180	100	0.60	5	4	2	180	100	0.60	5
23	4	2	95	100	0.60	5	4	2	95	100	0.60	5
24	4	1	95	100	0.60	5	4	1	95	100	0.60	5
(25)	4	1	360	100	0.60	2	4	1	360	100	0.60	2
26	4	1	180	100	0.60	2	4	1	180	100	0.60	2
27	4	1	95	100	0.60	2	4	1	95	100	0.60	2
(28)	4	2	95	100	0.60	10	4	2	95	100	0.60	10
(29)	4	2	180	100	0.60	5	4	2	95	100	0.60	5
(30)	4	1	360	100	0.60	2	4	1	360	100	0.60	2

The quantities represented by the symbols are defined as follows:

δp	the permanent speed droop setting in per cent
k	the effective speed droop signal expressed as a per cent of the permanent speed droop setting
V	the dashpot frequency signal expressed as a per cent of the effective speed droop signal
γ	the percentage load setting of the controller
T_w	the water starting time constant
m	the sampling duration in seconds

The bracketed tests are shown in Section III-2 of Chapter III.

Synopsis

The required system building blocks; generator, governor, hydro-system, load and controller models were simulated on the computer. The voltage regulator model, developed in Reference 4, was accepted and incorporated with the generator in Section II-1-6 of Chapter II.

The operation of the controller was not found to adversely affect the voltage variations in the generator and voltage regulator models for the sampling periods tested (2 to 10 seconds). Further it was found that the controllers did not affect the power oscillations between the generating units. However, it was found that the sampling periods, of the individual generators, should be equal to or greater than the natural period of oscillation of the governing loop (10 seconds) (20). The governing loop consists of the generator, turbine and the governor.

It was found that the controllers' signals, between generating units, should be related through the permanent speed droop curves. Further, it was found that the magnitude of the individual controller's signals, were dictated by the requirements for; a minimum system disturbance, maximum controller response and accurate load division among the generating stations.

Section I - 2 The Method of System Simulation

Two methods of system simulation are considered.

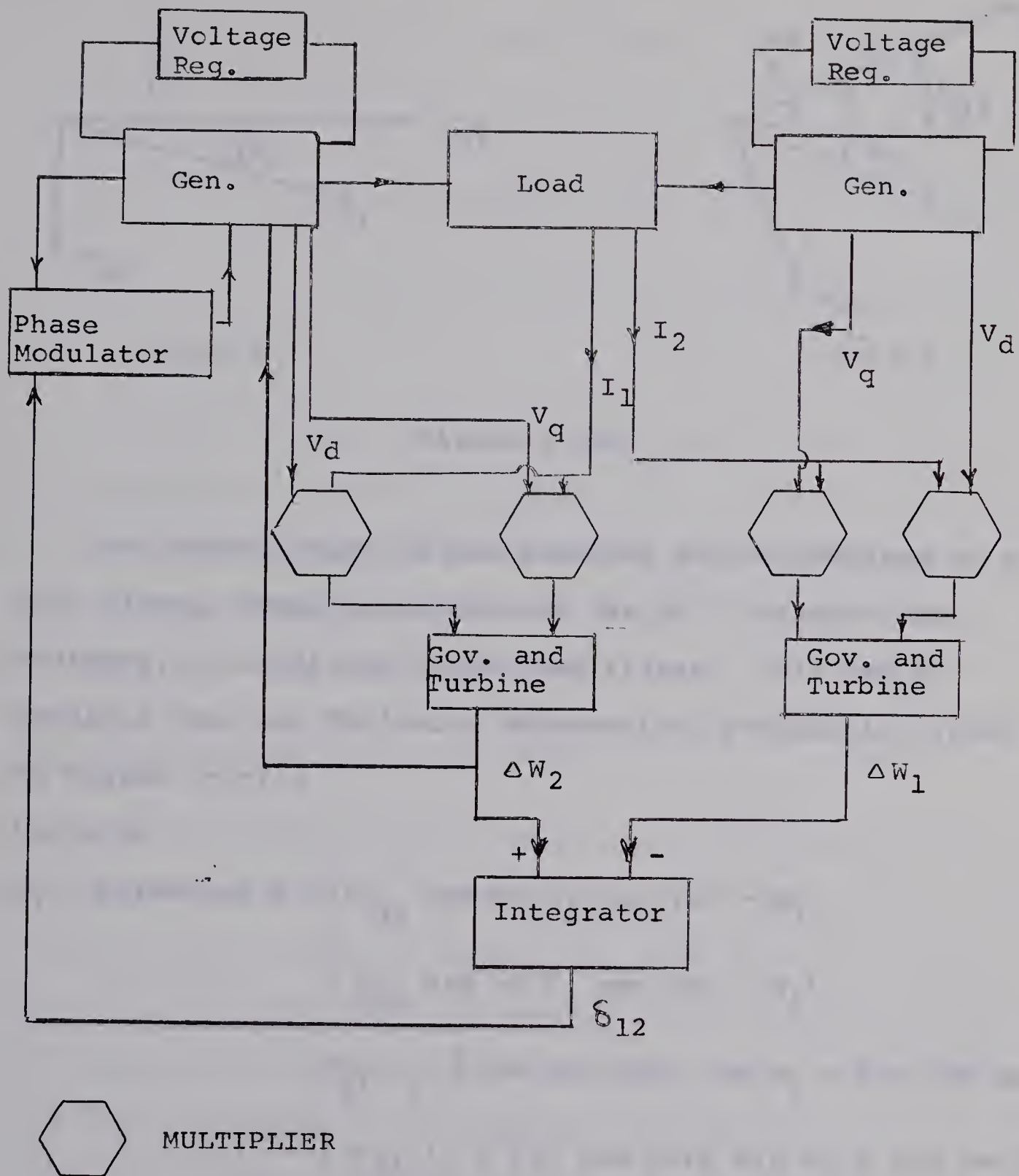
First, the representation of the generator, voltage regulator, governor and hydro-system on the analogue computer with the load being represented on the network analyzer. Second, the complete system is represented on the analogue computer.

Method I

The network analyzer operates on a frequency of 500 cycles per second. It is mainly used for transient stability studies where constant voltage behind the direct axis synchronous reactance and constant input power over the duration of the disturbance can be assumed. As the desired studies are of the steady state nature, it is necessary to automate the analyzer as the former assumptions are no longer valid. Therefore two possibilities exist: operate the system at 500 cycles or represent the steady state conditions by D.C. and convert between A.C. and D.C. when going from the computer to the analyzer.

A.C. - A.C. representation (refer to Figure I-2-I)

The unit's direct and quadrature axes currents and voltages are represented by A.C. quantities. The power output is a D.C. signal directly proportional to $V_q I_q + V_d I_d$. This is developed as follows:



A.C. - A.C. REPRESENTATION

Figure I-2-1

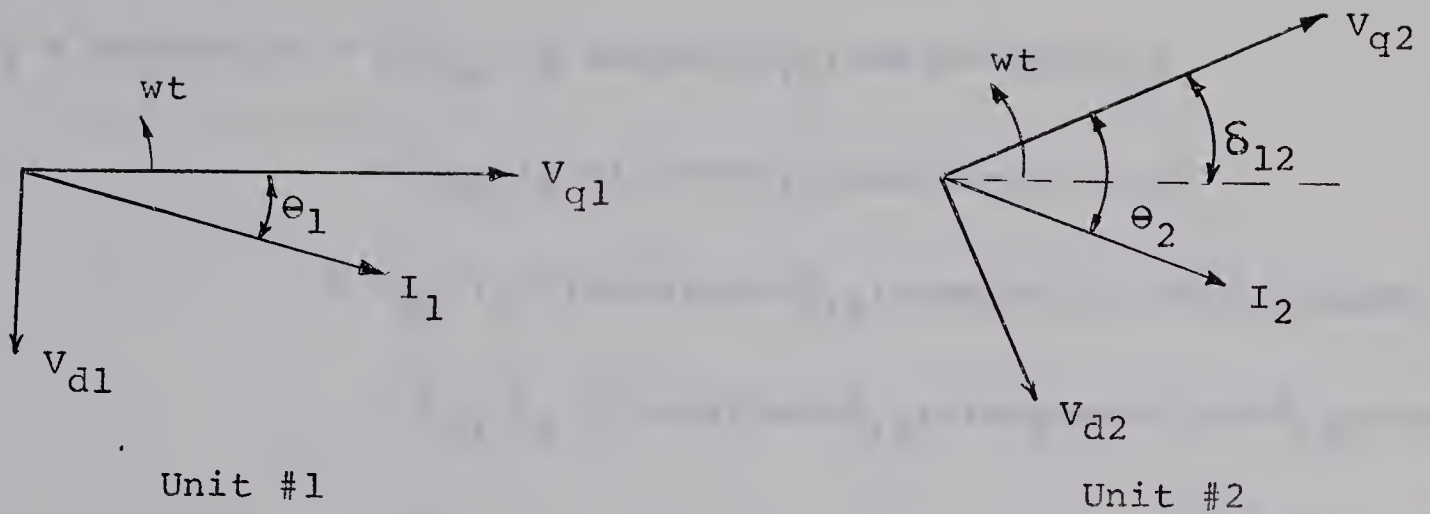


Figure I-2-2

The power output of the machines may be obtained as a D.C. signal, from the product of the A.C. currents and voltages, by utilizing a low pass filter. This may be realized from the following mathematical statements (refer to Figure I-2-2).

Unit #1

$$\begin{aligned}
 P_1 + \text{Harmonics} &= 2(V_{q1} \cos wt I_1 \cos (wt - \theta_1) \\
 &\quad + V_{d1} \sin wt I_1 \cos (wt - \theta_1)) \\
 &= V_{q1} I_1 [(1 + \cos 2wt) \cos \theta_1 + \sin 2wt \sin \theta_1] \\
 &\quad + V_{d1} I_1 [(1 + \cos 2wt) \sin \theta_1 + \sin 2wt \cos \theta_1]
 \end{aligned}
 \tag{I-2-1}$$

Unite #2

$$\begin{aligned}
 P_2 + \text{Harmonics} &= 2 [V_{q2} I_2 \cos(wt + \delta_{12}) \cos(wt - \theta_2 + \delta_{12}) \\
 &\quad + V_{d2} I_2 \sin(wt + \delta_{12}) \cos(wt - \theta_2 + \delta_{12})] \\
 &= V_{q2} I_2 [1 + \cos 2(wt + \delta_{12}) \cos \theta_2 + \sin 2(wt + \delta_{12}) \sin \theta_2] \\
 &\quad + V_{d2} I_2 [1 + \cos 2(wt + \delta_{12}) \sin \theta_2 + \sin 2(wt + \delta_{12}) \cos \theta_2]
 \end{aligned}$$

(I-2-2)

Utilizing a low pass filter one can eliminate the 1000 cycle components, producing equations 1-2-3 and 1-2-4, which are equal to the developed electrical power at the air gap when the equations are expressed in per unit quantities.

$$P_1 = V_{q1} I_1 \cos \theta_1 + V_{d1} I_1 \sin \theta_1 \quad (1-2-3)$$

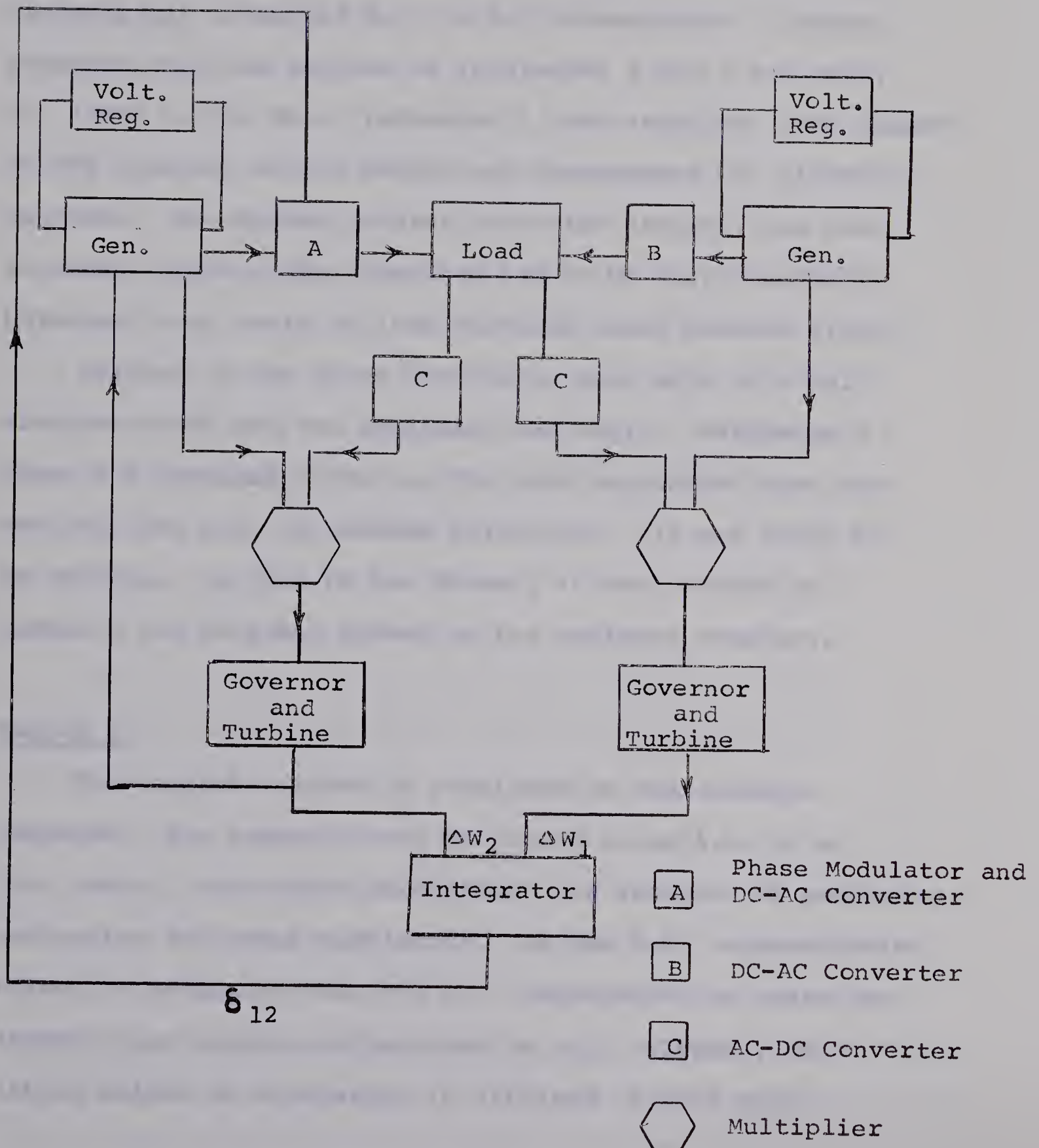
$$P_2 = V_{q2} I_2 \cos \theta_2 + V_{d2} I_2 \sin \theta_2 \quad (1-2-4)$$

Thus frequency, torque angle, input and output power are represented by D.C. quantities. It is necessary to phase modulate the voltage of either unit with the D.C. voltage depicting the synchronizing torque. The two major problems encountered in the above simulation are the phase modulation and the representation of generator saturation.

A.C. - D.C. Representation (refer to Figure II-1-3).

The portrayal of saturation may be included in the construction of a D.C. generator model. This approach also requires phase modulation and provides further problems as it is necessary to make A.C. to D.C. and D.C. to A.C. conversions.

Although no evidence of A.C. - A.C. simulations have



A.C. - D.C. REPRESENTATION

Figure I-2-3

been found in literature, references 1, 2 and 3 have successively attempted A.C. to D.C. conversions. Correspondence with the authors of references 2 and 3 was made. Dr. James E. Van Ness, reference 2, was sceptical with regard to the accuracy of his method and recommended Dr. Kimbark's approach. Dr. Kimbark stated that high accuracy had been attained, however the circuitry had to be very carefully balanced, thus tests of long duration could produce errors.

Neither of the above references were able to supply diagrams as to how the equipment was built. Reference 3 shows the required circuitry for both approaches when converting from D.C. to sixteen kilocycles. It was found to be complex. In view of the former, it was decided to simulate the complete system on the analogue computer.

Method II

The complete system is simulated on the analogue computer. The generator may be placed on an A.C. or a D.C. basis. The former encountered the problems of generator saturation and phase modulation. As the A.C. representation offers no advantage over the D.C. representation where the steady state values are depicted as D.C. voltages, the latter method of simulation is utilized in this study.

CHAPTER II

Section II-1 The Generator and Voltage Regulator Model

Section II-1-0 Introduction

The stator of the generator, under balanced three phase conditions can be portrayed on a per-phase basis by the use of Park's Equations. The rotor operates on a D.C. basis and the equations relating field voltage and current are known. The relation between the A.C. per-phase voltages and the D.C. voltage, applied to the rotor, are determined by the unit's voltage regulator. The voltage regulator equation, as derived in reference 4, was accepted as correct and utilized in this study. However, to obtain a simulation, it was still necessary to determine the rotor equations in terms of the stator equations.

The derivation of the equivalent per-phase stator circuit as seen by the stator and rotor is outlined in Sections II-1-1 and II-1-2. The relationship of the equivalent circuit as seen by both the stator and rotor are then developed in Section II-1-3.

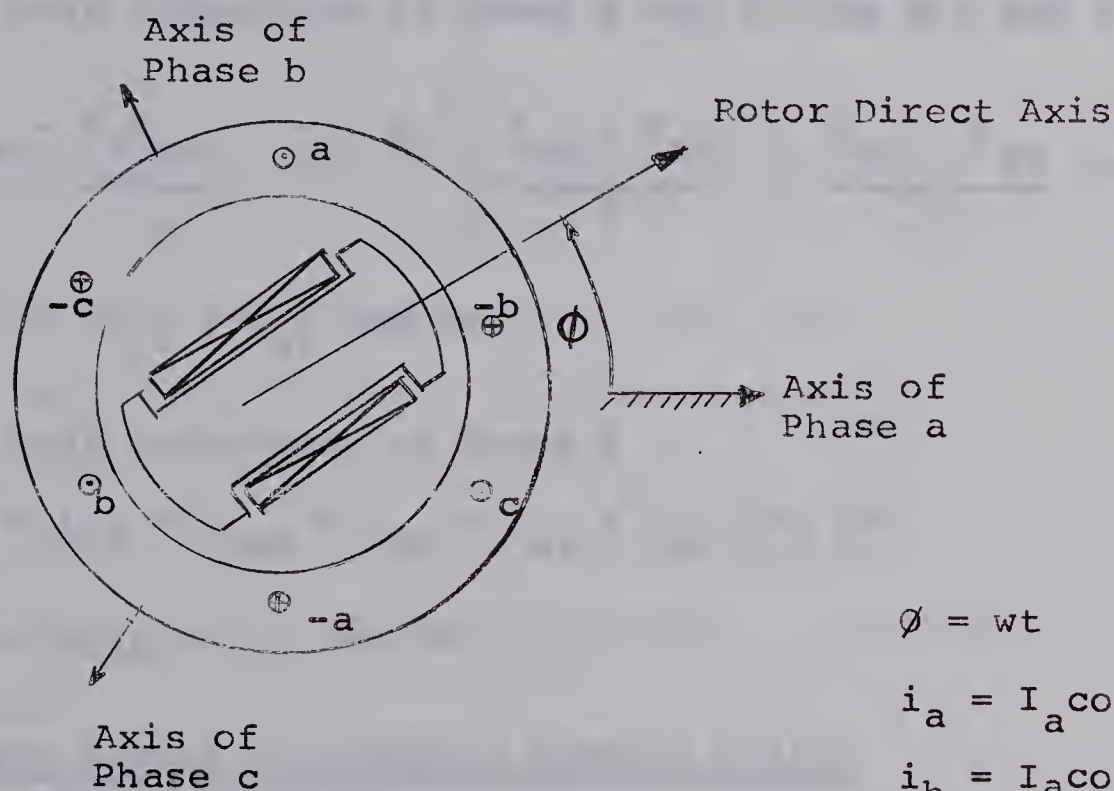
Motor action was assumed to be positive in the development of the equivalent circuits as the field and direct axis currents are in the same direction. This makes it unnecessary to attach a vector notation to them and simplifies the development. This assumption is accounted for in writing the equations for simulation in Section II-1-7.

The rotor parameters are transferred to the stator in Section II-1-4. The effect of generator saturation is

considered in Section II-1-5. The voltage regulator equation is stated in Section II-1-6. The equations representing the generator and voltage regulator are given in Section II-1-7.

Section II-1-1 The Stator Equivalent Circuit as Seen by the Stator Phase

The equations determining the stator and rotor self and mutual inductances are developed for phase A. (5)



$$\phi = \omega t$$

$$i_a = I_a \cos(\omega t - \theta)$$

$$i_b = I_a \cos(\omega t - 120^\circ - \theta)$$

$$i_c = I_a \cos(\omega t + 120^\circ - \theta)$$

Figure II-1-1

Stator Self Inductances (refer to Figure II-1-1)

Stator magnetomotive force

$$F_a = N_a i_a \quad (\text{II-1-1})$$

The force along the direct axis

$$F_{da} = F_a \cos \phi \quad (\text{II-1-2})$$

The force along the quadrature axis

$$F_{qa} = F_a \cos(\phi - 90^\circ) \quad (\text{II-1-3})$$

The flux created by the stator m.m.f.

$$\phi_{gda} = F_{da} P_{gd} = F_a P_{gd} \cos \theta \quad \text{along the direct axis} \quad (\text{II-1-4})$$

$$\phi_{gqa} = F_{qa} P_{gq} = -F_a P_{gq} \sin \theta \quad \text{along the quadrature axis} \quad (\text{II-1-5})$$

Flux linkages of phase A created by the air gap flux

$$\phi_{gaa} = \phi_{gda} \cos \theta - \phi_{gqa} \sin \theta \quad (\text{II-1-6})$$

$$= F_a \left[P_{gd} \cos^2 \theta + P_{gq} \sin^2 \theta \right] \quad (\text{II-1-7})$$

$$= N_a i_a \left[\frac{P_{gq} + P_{gd}}{2} + \frac{P_{gd} - P_{gq}}{2} \cos 2\theta \right] \quad (\text{II-1-8})$$

The self inductance of phase A due to the air gap flux

$$\begin{aligned} L_{gaa} &= \frac{N_a \phi_{gaa}}{i_a} = N_a^2 \left[\frac{P_{gd} + P_{gq}}{2} + \frac{P_{gd} - P_{gq}}{2} \cos 2\theta \right] \\ &= L_{g0} + L_{g2} \cos 2\theta \end{aligned} \quad (\text{II-1-9})$$

The self inductance of phase A

$$\begin{aligned} L_{aa} &= L_{a1} + L_{gaa} = L_{g0} + L_{a1} + L_{g2} \cos 2\theta \\ &= L_{a00} + L_{g2} \cos 2\theta \end{aligned} \quad (\text{II-1-10})$$

Stator mutual inductances between phases

The flux created by phase A

$$F_{da} P_{gd} = \phi_{gda} \quad \text{along the direct axis} \quad (\text{II-1-4})$$

$$F_{qa} P_{gq} = \phi_{gqa} \quad \text{along the quadrature axis} \quad (\text{II-1-5})$$

The flux created by phase A linking phase B

$$\phi_{gba} = \phi_{gda} \cos(\phi - 120) - \phi_{gqa} \sin(\phi - 120) \quad (\text{II-1-11})$$

$$\begin{aligned} \phi_{gba} &= F_a (P_{gd} \cos\phi \cos(\phi - 120) + P_{gq} \sin\phi \sin(\phi - 120)) \\ &= F_a \left[\frac{-P_{gd} + P_{gq}}{4} + \frac{P_{gd} - P_{gq}}{2} \cos(2\phi - 120) \right] \quad (\text{II-1-12}) \end{aligned}$$

The mutual inductance between phase A and phase B

$$L_{gba} = \frac{N_a \phi_{gba}}{i_a} = -0.5L_{go} + L_{g2} \cos(2\phi - 120) \quad (\text{II-1-13})$$

The mutual inductance of the rotor with phase A

Let the mutual inductance of the direct axis of the stator with the direct axis of the rotor to be L_{af} , then the mutual inductance with phase A is $L_{af} \cos\phi$. The rotor self inductance is then equal to L_{ff} , where $L_{ff} = L_{f1} + L_{af}$.

The remaining phase inductances may be determined in the same manner. The complete set of inductances are listed below:

$$L_{aa} = L_{aao} + L_{g2} \cos 2\phi$$

$$L_{bb} = L_{aao} + L_{g2} \cos(2\phi + 120)$$

$$L_{cc} = L_{aao} + L_{g2} \cos(2\phi - 120)$$

$$L_{aff} = L_{ffa} = L_{af} \cos\phi$$

$$L_{bff} = L_{ffb} = L_{af} \cos(\phi - 120)$$

$$L_{cff} = L_{ffc} = L_{af} \cos(\phi + 120)$$

$$L_{ab} = L_{ba} = -0.5L_{g0} + L_{g2} \cos(2\phi - 120)$$

$$L_{bc} = L_{cb} = -0.5L_{g0} + L_{g2} \cos(2\phi)$$

$$L_{ac} = L_{ca} = -0.5L_{g0} + L_{g2} \cos(2\phi + 120)$$

The per phase equations may be greatly simplified by transformation to the direct and quadrature axis. Thus equations II-1-14, II-1-15, II-1-16, II-1-17, and II-1-18 are defined.

$$F_{ds} = N_a [i_a \cos\phi + i_b \cos(\phi - 120) + i_c \cos(\phi + 120)] \quad (\text{II-1-14})$$

$$F_{qs} = N_a [-i_a \sin\phi - i_b \sin(\phi - 120) - i_c \sin(\phi + 120)] \quad (\text{II-1-15})$$

$$I_d = K_d [i_a \cos\phi + i_b \cos(\phi - 120) + i_c \cos(\phi + 120)] \quad (\text{II-1-16})$$

$$I_q = K_q [-i_a \sin\phi - i_b \sin(\phi - 120) - i_c \sin(\phi + 120)] \quad (\text{II-1-17})$$

$$F_{ds} = N_a I_d \quad F_{qs} = N_a I_q \quad (\text{II-1-18})$$

$$\frac{\quad}{K_d} \quad \frac{\quad}{K_q}$$

Letting $K_d = K_q = 2/3$, it is found that the following inverse transformation produces correct per phase currents.

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 1 \\ \cos(\phi - 120) & -\sin(\phi - 120) & 1 \\ \cos(\phi + 120) & -\sin(\phi + 120) & 1 \end{bmatrix} \begin{bmatrix} I_d \\ I_q \\ I_o \end{bmatrix} = \bar{i}_p = \bar{A} \bar{I}_q$$

Quadrature axis current transformation

$$\begin{bmatrix} I_d \\ I_q \\ I_o \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos\phi & \cos(\phi - 120) & \cos(\phi + 120) \\ -\sin\phi & -\sin(\phi - 120) & -\sin(\phi + 120) \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \bar{I}_q = \bar{B} \bar{i}_p$$

Per phase flux linkages, where I_{fs} and λ_{fs} are the rotor currents and flux linkages as seen by the stator.

$$\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_{fs} \end{bmatrix} = \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} & L_{aff} \\ L_{ab} & L_{bb} & L_{bc} & L_{bff} \\ L_{ac} & L_{bc} & L_{cc} & L_{cff} \\ L_{aff} & L_{bff} & L_{cff} & L_{ff} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_{fs} \end{bmatrix} = \bar{\lambda}_p = \bar{C} i_p$$

Quadrature axis flux linkage

$$\begin{bmatrix} \lambda_d \\ \lambda_q \\ \lambda_o \end{bmatrix} = \begin{bmatrix} \bar{B} \end{bmatrix} \begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \end{bmatrix} = \bar{\lambda}_q = \bar{B} \bar{\lambda}_p$$

Substituting the matrices $\bar{\lambda}_q = \bar{B} \bar{\lambda}_p$ and $\bar{i}_p = \bar{A} \bar{I}_q$ into the matrices $\bar{\lambda}_p = \bar{C} \bar{i}_p$ and expanding, the following equations are obtained:

$$\begin{aligned} \lambda_d &= I_{fs} L_{af} + I_d \left[L_{a1} + \frac{3}{2} (L_{go} + L_{g2}) \right] \\ &= I_{fs} L_{af} + I_d \left[L_{a1} + \frac{3}{2} N_a^2 P_{gd} \right] \\ &= I_{fs} L_{af} + I_d [L_{a1} + L_{ad}] \end{aligned} \quad (II-1-19)$$

$$\begin{aligned} \lambda_q &= I_q \left[L_{a1} + \frac{3}{2} (L_{go} - L_{g2}) \right] \\ &= I_q \left[L_{a1} + \frac{3}{2} (N_a^2 P_{gq}) \right] \\ &= I_q [L_{a1} + L_{aq}] \end{aligned} \quad (II-1-20)$$

$$\lambda_o = I_o L_{al} \quad (\text{II-1-21})$$

$$\lambda_{fs} = I_{fs} L_{ff} + \frac{3}{2} I_d L_{af} \quad (\text{II-1-22})$$

Expanding determinant $\bar{I}_q = \bar{B} \bar{i}_p$, it is found

$$I_d = I_a \sin \theta \quad (\text{II-1-23})$$

$$I_q = I_a \cos \theta \quad (\text{II-1-24})$$

$$I_o = 0$$

Thus it is seen that the stator-rotor mutual impedances as viewed from the stator phase are equal to $\frac{3}{2} N_a P_{gd}$ and $\frac{3}{2} N_a P_{gq}$ along the direct and quadrature axis respectively. This, however, does not establish a relationship between L_{ad} and L_{af} . To obtain this relationship, the direct and quadrature axis circuit as seen by the rotor is developed in Section II-1-2.

Section II-1-2 Stator equivalent circuit as seen by the rotor

The phasor currents and windings are 120 electrical degrees apart. This results in an m.m.f. travelling around the stator at synchronous speed. Under steady state conditions, the stator's m.m.f. appears stationary and of constant magnitude to the rotor as the latter also rotates at synchronous speed. This may be realized by referring to Figures II-1-1 and II-1-2.

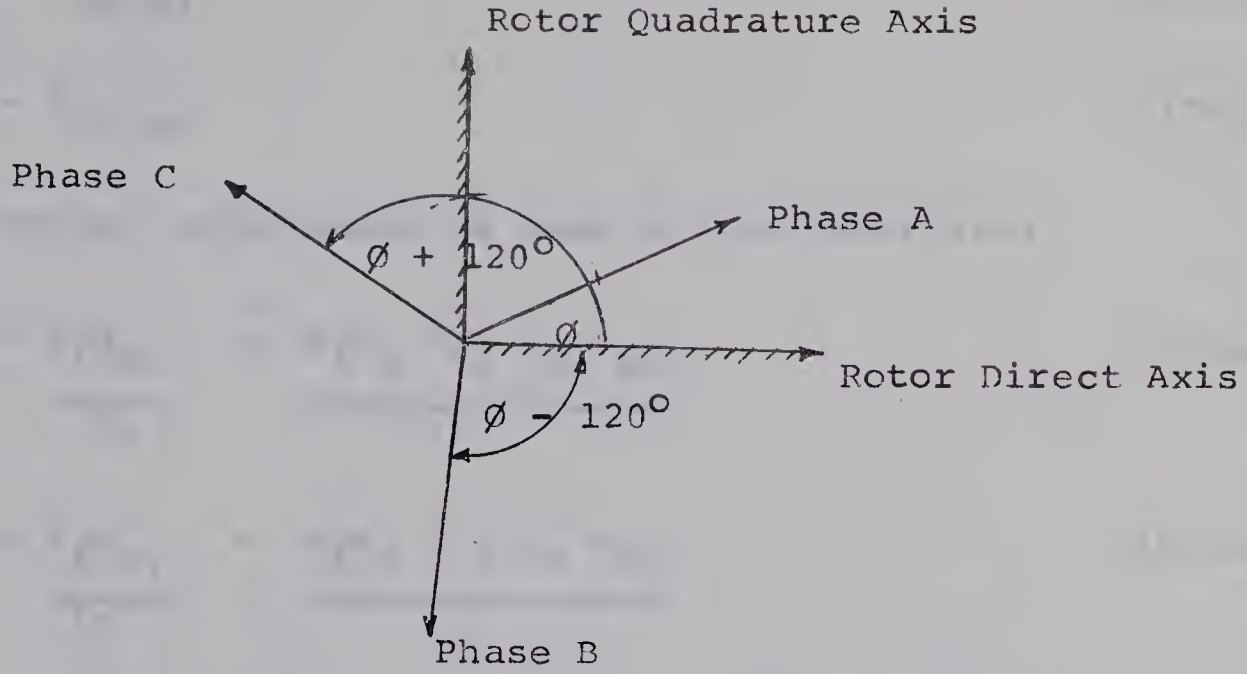


Figure II-1-2

The stator current components, referred to the stator, along the direct and quadrature rotor axes are $i_a \cos \phi$, $-i_a \sin \phi$, $i_b \cos(\phi - 120^\circ)$, $-i_b \sin(\phi - 120^\circ)$, $i_c \cos(\phi + 120^\circ)$ and $-i_c \sin(\phi + 120^\circ)$ respectively.

Expressing the relation in matrix form:

$$\begin{bmatrix} I'_d \\ I'_q \\ I'_o \end{bmatrix} = \begin{bmatrix} \cos \phi & \cos(\phi - 120^\circ) & \cos(\phi + 120^\circ) \\ -\sin \phi & -\sin(\phi - 120^\circ) & -\sin(\phi + 120^\circ) \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \bar{I}_q = \bar{D} \bar{i}_p$$

Expanding the determinant $\bar{I}_q = \bar{D} \bar{i}_p$

$$I'_d = \frac{3}{2} I_a \sin \theta = \frac{3}{2} I_d \quad (\text{II-1-25})$$

$$I'_q = \frac{3}{2} I_a \cos \theta = \frac{3}{2} I_q \quad (\text{II-1-26})$$

$$\text{Direct axis m.m.f.} \quad F_{ds} = \frac{3}{2} N_a I_d \quad (\text{II-1-17})$$

$$\text{Quadrature axis m.m.f.} \quad F_{qs} = \frac{3}{2} N_a I_q \quad (\text{II-1-18})$$

Flux created by the stator m.m.f.

$$\phi_{gd} = F_{ds} P_{gd} \quad (\text{II-1-27})$$

$$\phi_{gq} = F_{qs} P_{gq} \quad (\text{II-1-28})$$

The mutual inductances as seen by the rotor are:

$$L_{gd} = \frac{N_f \phi_{gd}}{I_f} = \frac{N_f N_a^{3/2} I_d P_{gd}}{I_f} \quad (\text{II-1-29})$$

$$L_{gq} = \frac{N_f \phi_{gq}}{I_f} = \frac{N_f N_a^{3/2} I_q P_{gq}}{I_f} \quad (\text{II-1-30})$$

As the stator and rotor are magnetically coupled, then a relationship $N_f I_f = \frac{3}{2} I_d N_a$ must exist. Thus equations II-1-29 and II-1-30 become:

$$L_{gd} = N_f^2 P_{gd} = (N_f/N_a)^2 \frac{2}{3} L_{ad} \quad (\text{II-1-31})$$

$$L_{gq} = N_f^2 P_{gq} = (N_f/N_a)^2 \frac{2}{3} L_{aq} \quad (\text{II-1-32})$$

The equations as viewed from the rotor

$$\lambda_{dr} = \frac{I_f}{N_f} L_{gd} + N_a \left(\frac{3}{2} I_d \right) L_{gd} \quad (\text{II-1-33})$$

$$\lambda_{qr} = \frac{N_a}{N_f} \left(\frac{3}{2} I_q \right) L_{gq} \quad (\text{II-1-34})$$

$$\lambda_o = 0$$

$$\lambda_f = \frac{I_f}{N_f} (L_{f1} + L_{gd}) + N_a \left(\frac{3}{2} I_d \right) L_{gd} \quad (\text{II-1-35})$$

Substituting equations II-1-31 and II-1-32 into II-1-33

to II-1-35, equations II-1-36 to II-1-38 are obtained.

$$\lambda_{dr} = \left[\frac{N_f}{N_a} \right]^2 \frac{2}{3} I_f L_{ad} + \frac{N_f}{N_a} I_d L_{ad} \quad (\text{II-1-36})$$

$$\lambda_{qr} = \frac{N_f}{N_a} I_q L_{aq} \quad (\text{II-1-27})$$

$$\lambda_o = 0$$

$$\lambda_f = I_f \left[L_{f1} + \left[\frac{N_f}{N_a} \right]^2 \frac{2}{3} L_{ad} \right] + \frac{N_f}{N_a} I_d L_{ad} \quad (\text{II-1-38})$$

Section II-1-3 The comparison of the generator's equations as seen from the stator phase and rotor

The rotor flux linkage equations will first be transferred to the stator, via the turns ratio (N_a/N_f), then the stator equations will be re-written to facilitate the comparison.

Rotor Flux Linkage Equations Transferred to the Stator

$$\lambda_d = N_a \frac{\lambda_{dr}}{N_f} = N_a \frac{2}{3} I_f L_{ad} + I_d L_{ad} \quad (\text{II-1-39})$$

$$\lambda_q = N_a \frac{\lambda_{qr}}{N_f} = I_q L_{aq} \quad (\text{II-1-40})$$

$$\lambda_o = 0$$

$$\lambda_{fs} = N_a \frac{\lambda_f}{N_f} = N_f \frac{2}{3} I_f \left[\frac{3}{2} \left[\frac{N_a}{N_f} \right]^2 L_{f1} + L_{ad} \right] + I_d L_{ad} \quad (\text{II-1-41})$$

Stator Flux Linkage Equations

$$\lambda_d = I_{fs} L_{af} + I_d (L_{al} + L_{ad}) \quad (\text{II-1-19})$$

$$\lambda_q = I_q (L_{al} + L_{aq}) \quad (\text{II-1-20})$$

$$\lambda_o = 0$$

$$\lambda_f = I_{fs} L_{ff} + \frac{3}{2} I_d L_{af} \quad (\text{II-1-22})$$

Comparing the equations, it may be seen that $\frac{3}{2} L_{af} = L_{ad}$

$$L_{ff} = L_{ad} + \frac{3}{2} (N_a/N_f)^2 L_{fl}$$

$$I_{fs} = (N_f/N_a)^{2/3} I_f$$

Further, it may be realized that Park's equations divide the stator's direct and quadrature currents and multiply the impedance by a factor of $\frac{3}{2}$. As it is desired to represent the generator on a per phase basis, this factor will be applied to the field quantities.

Since the stator quantities will be represented on a per-phase basis, it is necessary to establish the stator-rotor turns ratio to determine the rotor base quantities.

Section II-1-4 The Rotor Per Unit Quantities

The rotor and stator are coupled together through a mutual inductance L_{ad} , which may be represented pictorially together with the respective leakage reactances, as shown in Fig. II-1-3.

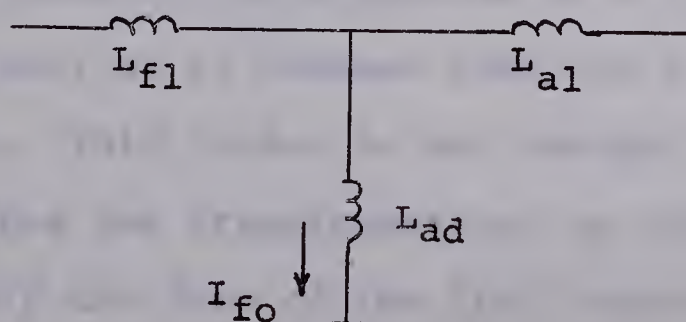


Figure II-1-3

The current I_{fo} may be considered as the magnetizing current required to produce Rated Voltage on the Air-Gap line as shown in Figure II-1-4.

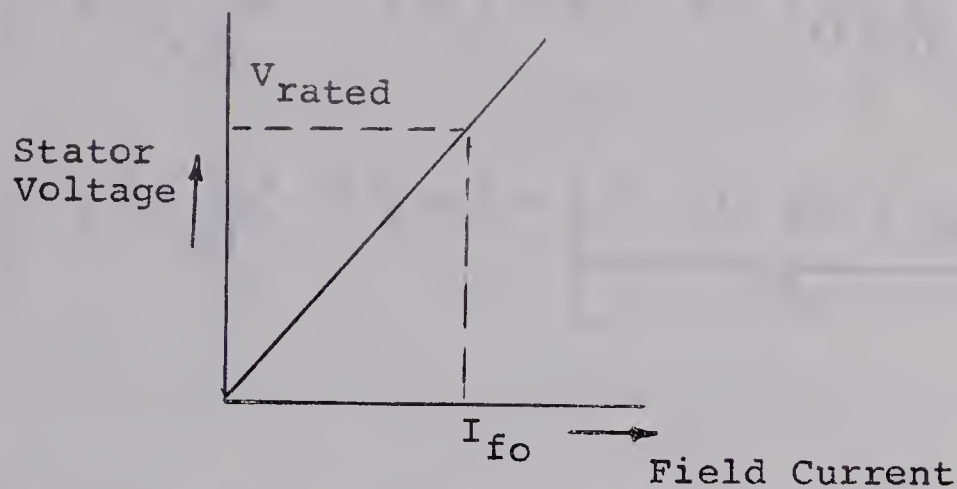


Figure II-1-4

Thus in accordance with the turns ratio established in Section II-1-3

$$\left(\frac{2}{3} I_{fo} N_f / N_a\right) X_{ads} = V_{\text{peak/phase}} \quad (\text{II-1-42})$$

$$\therefore N_a / N_f = a = \frac{V_{\text{peak/phase}}}{\frac{2}{3} I_{fo} X_{ads}} \quad (\text{II-1-43})$$

Therefore the base rotor quantities become

$$V_{Br} = \frac{V_{\text{peak/phase}}}{a} \quad (\text{II-1-44})$$

$$I_{Br} = a I_{\text{peak/phase}} \quad (\text{II-1-45})$$

This transformation results in a fictitious base rotor field voltage, as it assumes that the rotor operates on a A.C. basis. This factor is not thought to be significant however since the transformation, as will now be shown, affects only the form of the field equations.

$$V_f = I_f R_f + L_{ff} \frac{d I_f}{dt} \quad (\text{II-1-46})$$

$$aV_f = \left[\frac{2}{3} I_f / a \right] \left[a^2 \frac{3}{2} R_f \right] + \left[a^2 \frac{3}{2} L_{ff} \right] \frac{d}{dt} \left[\frac{2}{3} I_f / a \right]$$

$$aV_f = \left[\frac{2}{3} I_f / a \right] \left[a^2 \frac{3}{2} R_f \right] + \left[\frac{a^2 \frac{3}{2} X_{f1} + X_{ads}}{W_o} \right] \frac{d \left[\frac{2}{3} I_f / a \right]}{dt}$$

Expressing the equation in per unit

$$\frac{aV_f}{V_{\text{peak/phase}}} = \frac{\left[\frac{2}{3} I_f / a \right]}{I_{\text{peak/phase}}} \frac{\left[a^2 \frac{3}{2} R_f \right]}{X_{Bs}} + \frac{\left[a^2 \frac{3}{2} X_{f1} + X_{ads} \right]}{W_o X_{Bs}} \frac{d}{dt} \frac{\frac{2}{3} I_f / a}{I_{\text{peak/phase}}}$$

$$W_o V_f = (I_f W_o R_f) + (X_1 + X_{ads}) \frac{d}{dt} (I_f) \quad (\text{II-1-47})$$

$$\text{Where } I_f \text{ p.u.} = \frac{\frac{2}{3} I_f / a}{I_{\text{peak/phase}}}$$

$$R_f \text{ p.u.} = \frac{a^2 \frac{3}{2} R_f}{X_{Bs}}$$

$$X_{f1} \text{ p.u.} = \frac{a^2 \frac{3}{2} X_{f1}}{X_{Bs}}$$

Section II-1-5 Generator Saturation

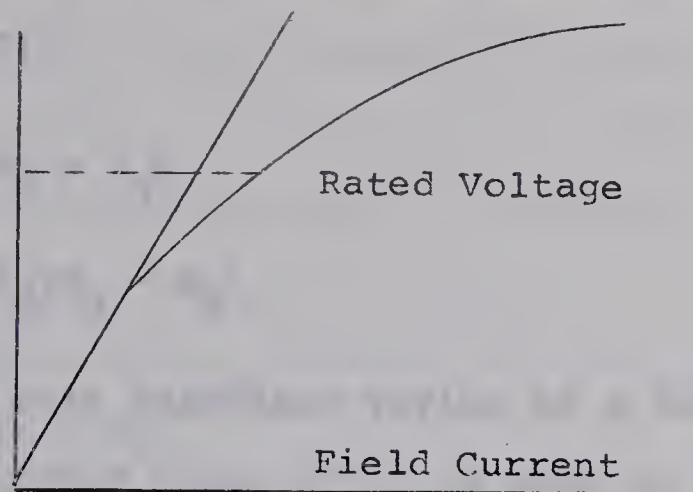


Figure II-1-4

The voltage behind the armature leakage reactance varies along the open circuit curve in the region near rated voltage. Thus in this range of voltages, the deviation between the o.c.c. and the air gap line is not large (Figure II-1-6), therefore the former voltage is assumed to vary along the air gap line.

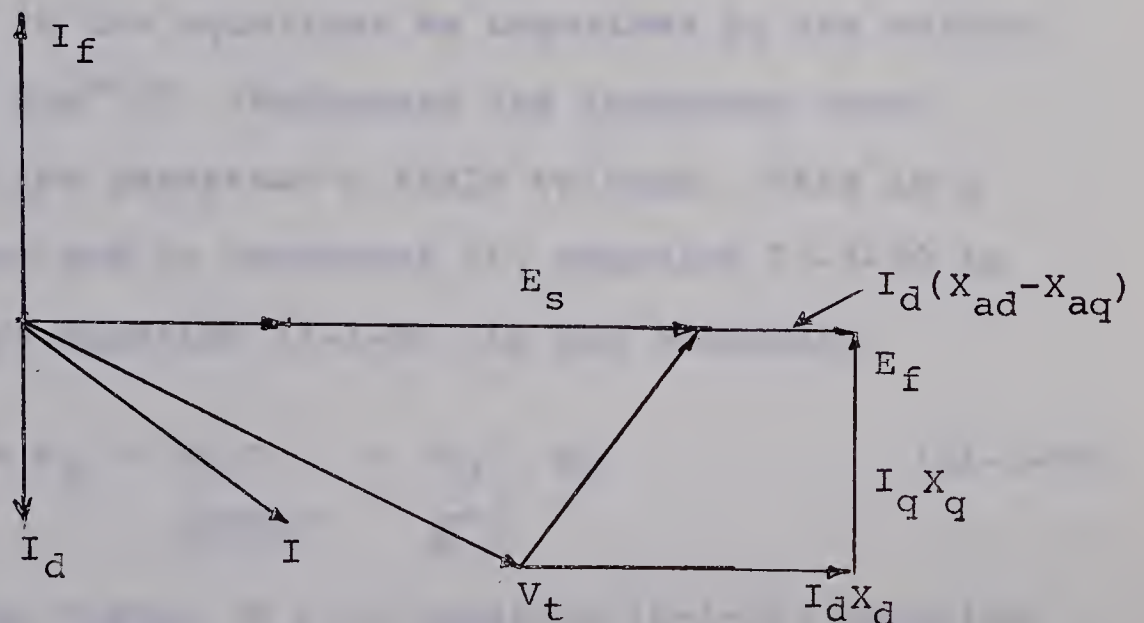


Figure II-1-7

It is necessary to obtain the voltage behind the quadrature reactance to represent the salient pole machine (6) (Figure II-1-7)

$$E_q = E_f - I_d(X_d - X_q)$$

$$E_q = I_f X_{ad} - I_d(X_d - X_q) \quad (\text{II-1-48})$$

As the direct axis reactance varies as a function of the unit's load, the saturated value of X_d is used in the above equation.

Section II-1-6 The Voltage Regulator

Equation II-1-49, describing the voltage regulating system in per unit values, is obtained from the thesis presented by O. W. Hanson, reference 4.

$$V'_f = K_1 E_R e^{-t/T} + K_2 E_R + K_3 \int E_R dt \quad (\text{II-1-49})$$

where $E_R = V_{act} - V_{ref}$, $K_1 = 1$, $K_2 = 5$, $K_3 = 0.2$, $T = 10$

Due to the difference in the method of simulation, V'_f is equal to $W_O V_f$ in the equations as described by the author.

The term $E_R e^{-t/T}$ represents the temporary over-regulation of the generator's field voltage. This is a non-linear term and to represent it, equation II-1-49 is approximated by equation II-1-50 (in the s-domain).

$$W_O V_f = \left[\begin{array}{c} (K_1 + K_2) - \frac{K_1/T}{s+1/T} + \frac{K_3}{s} \end{array} \right] E_R \quad (\text{II-1-50})$$

Applying a step change (E_r) in equation II-1-50, equation II-1-49 is obtained. As the system is tested by applying

step load changes, which approximately result in step voltage changes, the latter equation is a valid approximation.

It is noted, in the reference's computer schematic, that this term ($E_r e^{-t/T}$) is represented as a delay. It is further noted that Mr. Hanson attained 5% agreement between his computer simulation and field results.

On the above basis, equation II-1-50 is accepted and utilized in this study.

Section II-1-7 The Generator and Voltage Regulator Equations

The damper windings are located on the rotor; therefore they have the same appearance to the stator as does the rotor circuit. The equations are now expressed in per unit and written for generator action.

$$\lambda_d = I_f L_{ad} - I_d(L_{al} + L_{ad}) + I_{kd}L_{ad} \quad (7) \quad (II-1-51)$$

$$\lambda_q = I_{kq}L_{aq} - I_q(L_{al} + L_{aq}) \quad (II-1-52)$$

$$\lambda_{kd} = I_f L_{ad} - I_d L_{ad} + I_{kd}(L_{ad} + L_{kd}) \quad (II-1-53)$$

$$\lambda_{kq} = I_{kq}(L_{aq} + L_{kq}) - I_q L_{aq} \quad (II-1-54)$$

$$V_q = W\lambda_d + s\lambda_q - I_q R_a \quad (II-1-55)$$

$$V_d = -W\lambda_q + s\lambda_d - I_d R_a \quad (II-1-56)$$

$$V_{kd} = I_{kd} R_{kd} + s\lambda_{kd} = 0 \quad (II-1-57)$$

$$V_{kq} = I_{kq} R_{kq} + s\lambda_{kq} = 0 \quad (II-1-58)$$

$$V_f = I_f R_f + s\lambda_f \quad (II-1-59)$$

$$W_o V_f = E_R \left[\frac{(K_1 + K_2) - K_1/T}{s + 1/T} + K_3 \right] \quad (\text{II-1-50})$$

$$E_q = I_f X_{ad} - I_d [X_d - X_q] + I_{kd} X_{ad} \quad (\text{II-1-60})$$

$$T_u = \lambda_q I_q - \lambda_d I_d \quad (9) \quad (\text{II-1-61})$$

$$V_t = \sqrt{V_d^2 + V_q^2} \quad (\text{II-1-62})$$

Approximations made to the above equations

1. The terms $s\lambda_q$ and $s\lambda_d$ are neglected as the resulting error is less than one per cent. (9)
2. The armature resistance is considered to be negligible.

It is noted that the term $I_{kd} X_{ad}$ is added to equation II-1-48 to produce equation II-1-60 as it acts to hold the voltage behind the quadrature reactance constant.

With these approximations, the following equations are derived from equations II-1-50 to II-1-62 and used for the analogue simulation.

$$\begin{aligned} W_o \lambda_f &= W_o (V_f - I_f R_f) / s \\ &= I_f (X_{ad} + X_{f1}) - I_d X_{ad} + I_{kd} X_{ad} \end{aligned} \quad (\text{II-1-63})$$

$$\begin{aligned} E_{gp} &= W_o \lambda_f - I_f X_{f1} \\ &= (I_f - I_d + I_{kd}) X_{ad} \end{aligned} \quad (\text{II-1-64})$$

$$W_o \lambda_d = E_{gp} - I_d X_{a1} \quad (\text{II-1-65})$$

$$V_q = W \lambda_d \quad (\text{II-1-66})$$

$$I_{kd} = \frac{W_o \lambda_{kd} - E_{gp}}{X_{kd}} \quad (\text{II-1-67})$$

$$\begin{aligned}
 W_o \lambda_{kd} &= \frac{W_o (V_{kd} - I_{kd} R_{kd})}{s} \\
 &= \frac{-W_o I_{kd} R_{kd}}{s}
 \end{aligned}
 \tag{II-1-68}$$

$$W_o \lambda_q = -I_q (X_{aq} + X_{al}) + I_{kq} X_{aq} \tag{II-1-69}$$

$$V_d = -W \lambda_q \tag{II-1-70}$$

$$I_{kq} = \frac{W_o \lambda_{kq} + I_q X_{aq}}{X_{aq} + X_{kq}} \tag{II-1-71}$$

$$\begin{aligned}
 W_o \lambda_{kq} &= \frac{W_o (V_{kq} - I_{kq} R_{kq})}{s} \\
 &= \frac{-I_{kq} W_o R_{kq}}{s}
 \end{aligned}
 \tag{II-1-72}$$

$$W_o V_f = E_R \left[\frac{(K_1 + K_2) - K_1/T + K_3}{s+1/T} \right] \tag{II-1-50}$$

$$E_q = I_f X_{ad} - I_d [X_d - X_q] + I_{kd} X_{ad} \tag{II-1-60}$$

$$P_L = WT_u = V_q I_q + V_d I_d \tag{8} \tag{II-1-73}$$

$$V_t = \sqrt{V_d^2 + V_q^2} \tag{8} \tag{II-1-74}$$

Section II-1-8 Conclusion

It was found that Park's equations, which represent the stator on a per-phase basis, reduce the current in the rotor and increase the rotor impedances by a factor of $3/2$. This multiplication factor does not affect the equations (stator or rotor) as it leaves the flux linkages unaltered.

As it is desired to represent the generator on a per-phase basis, the above multiplier was applied to the respective rotor quantities. They were expressed in terms of the stator equations through the turns ratio of the generator rotor and stator.

Section II-2 The Speed Governor Model

Section II-2-0 Introduction

The governor and controller determine the turbine wicket gate position as a function of speed. The speed of the unit varies as the integral of the difference between the power input and power output of the system. (refer to Figure II-2-1)

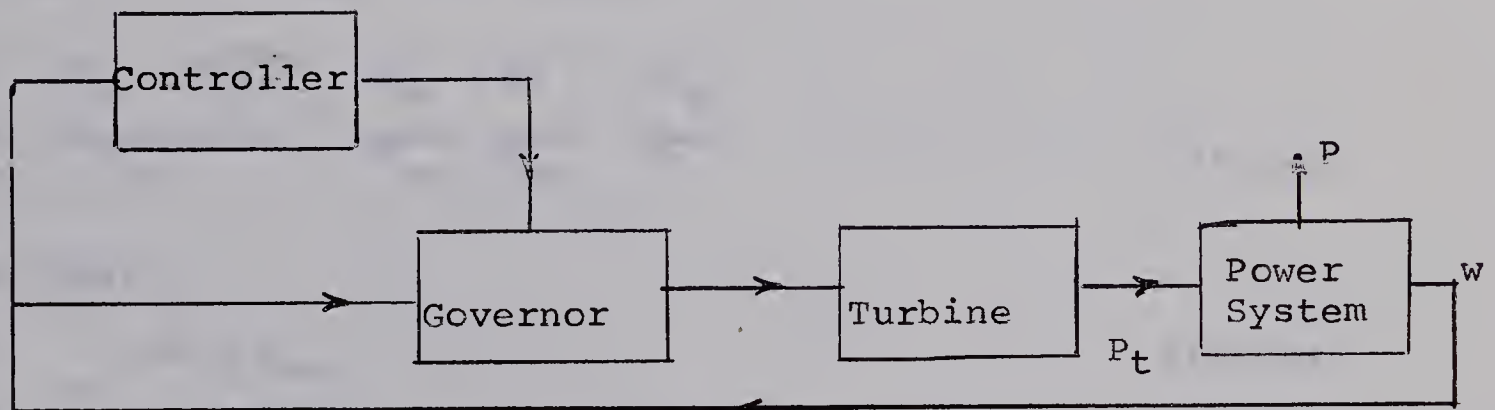


Figure II-2-1

The relation between the power and speed deviation is developed in Section II-2-1. The governor transfer function, relating the speed deviation to the wicket gate position is developed in Section II-2-2. The controller action is superimposed in Chapter II, Section II-5-1.

Section II-2-1 The Power-Speed Relationship

The unit's speed varies as a function of the difference between the turbine and load torque.

$$T_T - T_L = \frac{WR^2}{g} \frac{d(W_m)}{dt} \quad (\text{II-2-1})$$

$$\frac{T_T - T_L}{T_{rr}} = \frac{(2/p)WR^2}{T_{rr}g} \frac{d(2\pi f_e)}{dt}$$

$$\text{Assuming Rated KVA} = \text{Rated KW} = \frac{W_{mo} T_{rr}}{737.6} \quad (\text{II-2-2})$$

$$\frac{T_T - T_L}{T_{rr}} = \frac{WR^2}{g} \frac{W_{mo}^2}{737.6 \text{ KW } (p/2W_{mo})} \frac{d(2\pi f_e)}{dt}$$

$$\frac{T_T - T_L}{T_{rr}} = \frac{2H}{2\pi f_{eo}} \frac{d(2\pi f_e)}{dt}$$

$$\frac{W_m}{W_{mo}} \frac{(T_T - T_L)}{T_{rr}} = \frac{f_e}{f_{eo}} \frac{2H}{f_{eo}} \frac{df_e}{dt} \quad (\text{II-2-3})$$

Note that

$$\frac{f_e}{f_{eo}} = \frac{\Delta f + f_{eo}}{f_{eo}} \quad (\text{II-2-4})$$

$$\frac{\Delta f_e}{f_{eo}} = \frac{\Delta W_m}{W_{mo}} = \Delta f \text{ p.u.} \quad (\text{II-2-5})$$

Therefore equation II-2-3 becomes

$$\frac{T_T - T_L}{T_{rr}} \Delta f \text{ p.u.} = \Delta f \text{ p.u.} \frac{2H}{f_{eo}} \frac{d(\Delta f \text{ p.u.})}{dt} \quad (\text{II-2-6})$$

Expressing equation II-2-2 in per unit

$$P_T - P_L = f \cdot 2H \frac{df}{dt} \quad (\text{II-2-7})$$

The turbine and load power are functions of system frequency, refer to Figure II-2-2 which shows their variations when the input power to the turbine is held constant. (10)

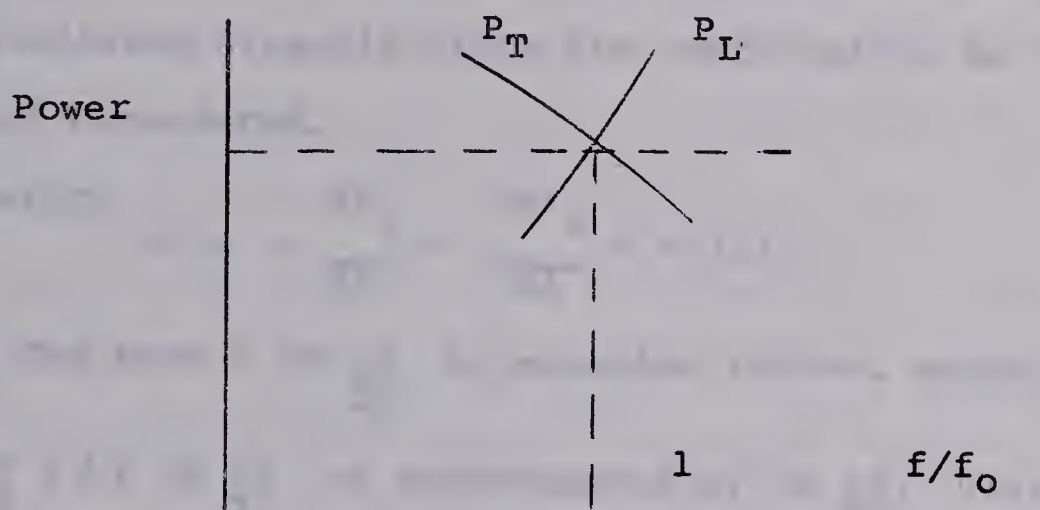


Figure II-2-2

Expanding P_T and P_L in a truncated Taylor series about synchronous speed equation II-2-8 is obtained which is expressed in per unit quantities.

$$P_t - P_l + \left[\frac{dP_t}{df} - \frac{dP_l}{df} \right] df = f \cdot 2H \frac{df}{dt} \quad (\text{II-2-8})$$

The magnitude of the term $\frac{dP_t}{df}$ is contingent on the synchronous speed of the turbine. For low and high synchronous speeds, its values are approximately equal to -1.1 and -0.6 respectively.

The magnitude of the term $\frac{dP_l}{df}$ depends on the nature of the load and the voltage frequency relationship of the voltage regulator. The load contribution to the above term

varies between 0 and 2 depending on wheather the load is principally ohmic resistance or turbo-machines, respectively.

In the simulation, the damping action of the load is absorbed in the term $\frac{dP_1}{df}$ and then the load is considered

to be independent of frequency. The voltage regulator, however, is simulated directly, thus its contribution to the term $\frac{dP_1}{df}$ is not considered.

Therefore

$$-0.6 > \frac{dP_t}{df} - \frac{dP'_1}{df} > -3.1$$

The term $f 2H \frac{df}{dt}$ in equation II-2-8, which is equal to

$2H \frac{df}{dt} + \Delta f 2H \frac{df}{dt}$, is approximated by $2H \frac{df}{dt}$. This is justified

as $\Delta f 2H \frac{df}{dt}$ is small when considering speed deviations of

less than .04 per unit. Small deviations in speed are

considered as it is desired to analyze the system under

normal operating conditions. Thus equation II-2-8 becomes

$$(P_t - P'_1) - P_d \Delta f = 2H \frac{df}{dt} \quad (\text{II-2-9})$$

Where

$$P_d = \frac{dP_t}{df} - \frac{dP'_1}{df}$$

P_t is the developed turbine power at rated speed

P'_1 is the load output power at rated speed plus the change in power resulting from the voltage regulator frequency relationship.

$\frac{dp_t}{df}$ is the self regulation of the turbine

$$\frac{dp'_1}{df}$$
 is the self regulation of the load

Section II-2-2 The Governor Transfer Function

The governor's function is to hold the unit at synchronous speed. It accomplishes this by controlling the wicket gate position, and therefore the developed power, as a function of speed. The operation of the governor as depicted in reference II, is shown in Figure II-2-4. Simplification of the governor is then made to the extent shown in Figure II-2-6. In the development of the equations, the wicket gate movement is assumed to be slow enough that the governor is capable of sustaining its oil pressure. The block diagram of the governor action is shown in Figure II-2-3.

The sensing and detection of speed error is achieved in the ball head assembly by comparing the centrifugal (11) force of the flyweights against the force exerted by the

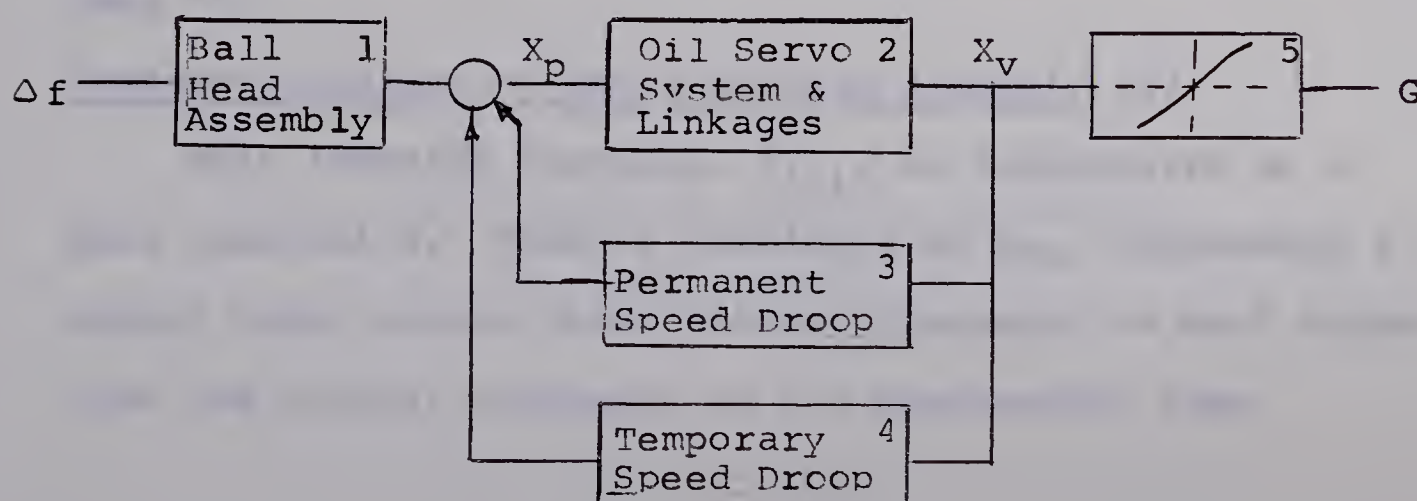
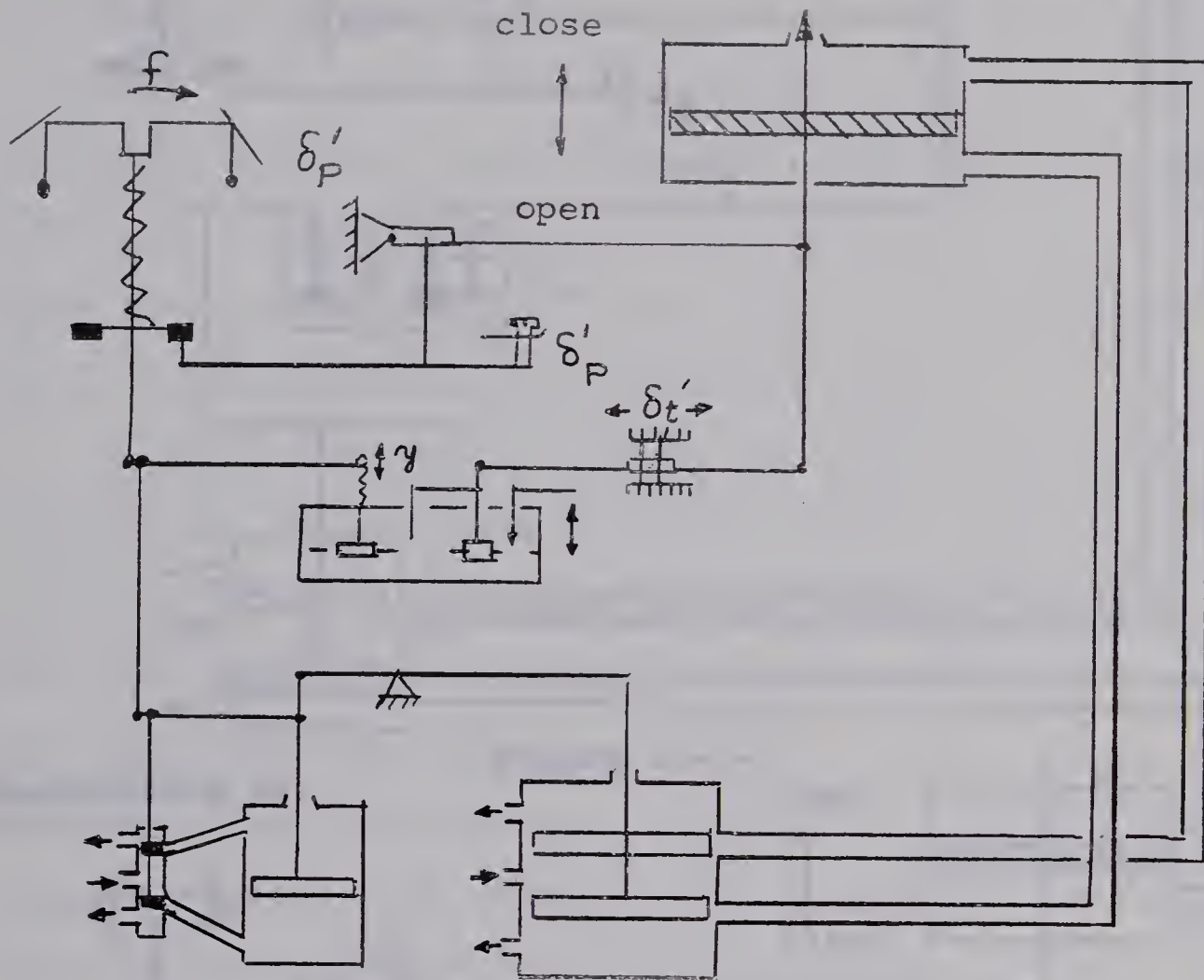


Figure II-2-3

speeder spring (Block #1). The movement of the flyweights is transmitted to the turbine wicket gates through mechanical linkages, pistons, valves and an oil pressure system (Block #2). The stabilizing features are represented in Blocks 3 and 4. Block #3 represents the permanent speed droop. This is the proportional reduction of the governor speed setting in a given ratio as the turbine gate opening increases. It is effected through the restoring mechanism and is required to produce stable load sharing between units. As the permanent speed droop is decreased the load change increases for a given speed change. Block #4 represents the temporary speed droop. Due to the water inertia, a delay is incurred between the wicket gate opening and the developed turbine power. As the former is proportional to the integral of the pilot valve position, it is necessary to provide a large permanent droop or another means of compensation to counteract over speed regulation. Therefore compensation is provided in the form of temporary speed droop. This results in an initial reduction of the governor's speed setting which gradually decreases to zero. Block #5 is discussed on page 39.

Transfer Function of the Ball Head Assembly (13)

This transfer function, $B(s)$, is represented as a gain constant B . This is justified as $B(s)$ represents a second order system whose natural frequency is much higher than the natural frequency of the governing loop.



THE HYDRAULIC GOVERNOR SCHEMATIC

Figure II-2-4

Simplified to:

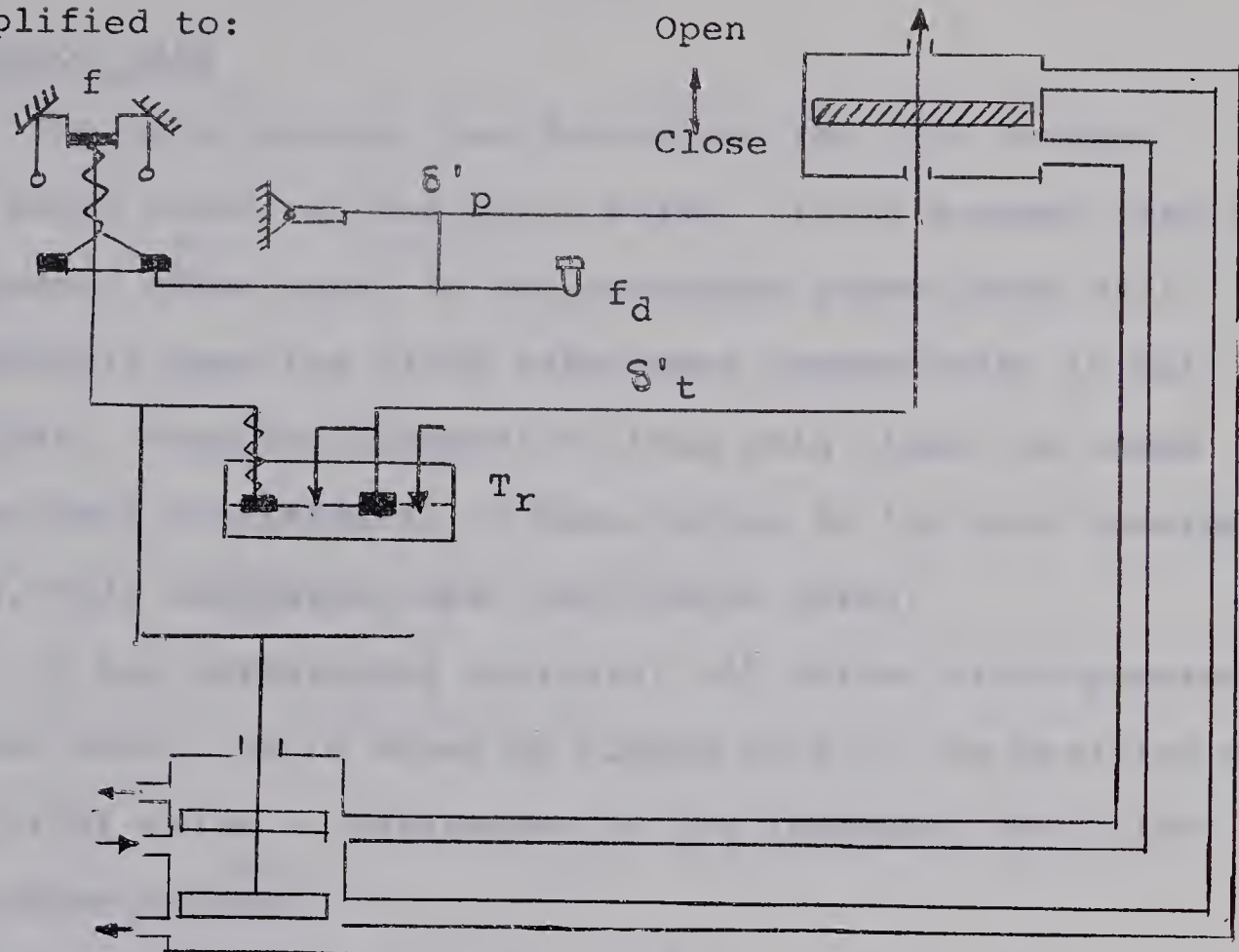
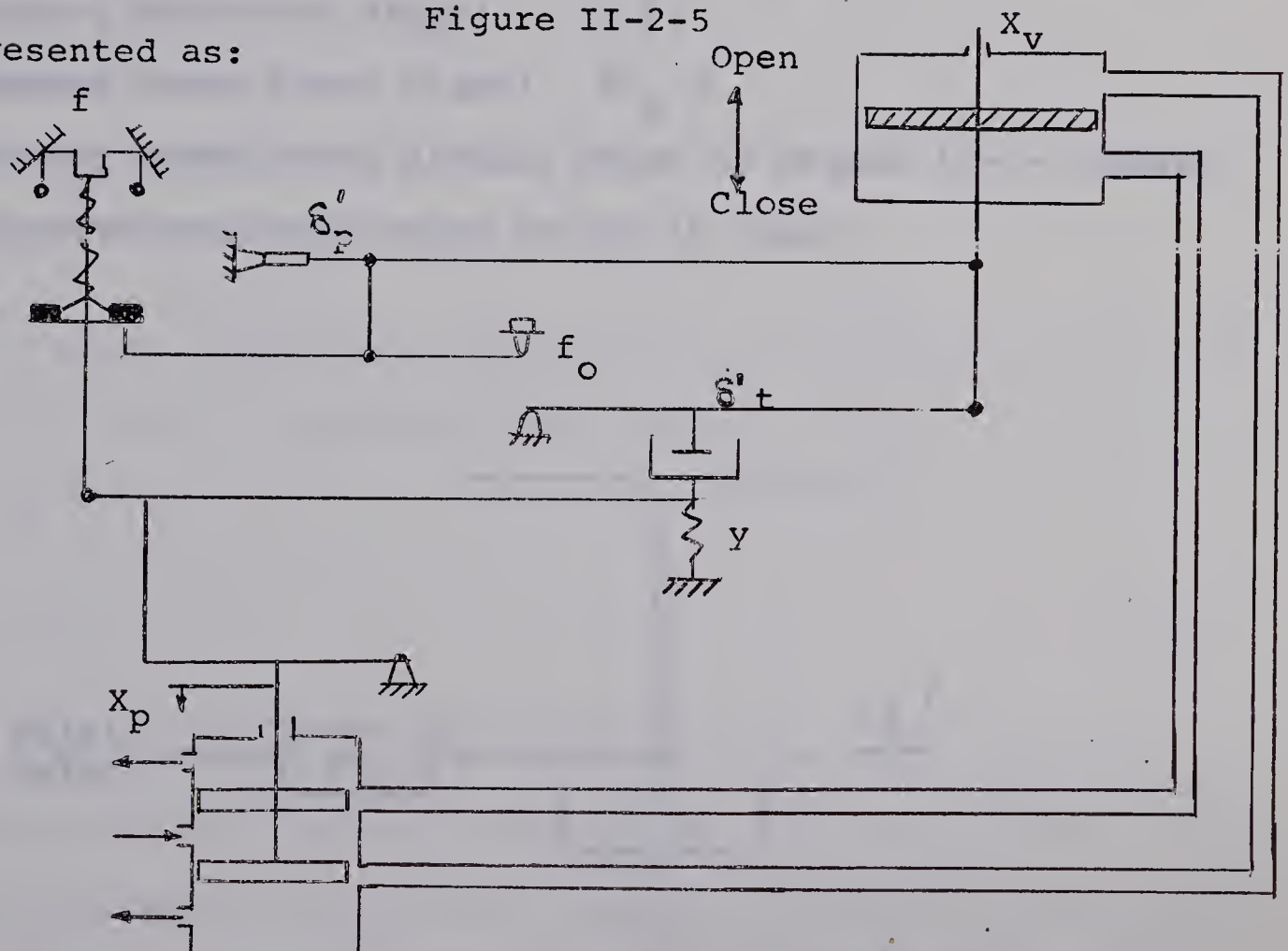


Figure II-2-5

Represented as:



SIMPLIFIED SCHEMATIC OF THE GOVERNOR

Figure II-2-6

Governor Gain

The gate opening time determines the gain between the servo motor and the pilot valve. It is assumed that a frequency error equal to the permanent speed droop will completely open the pilot valve when compensation is not applied. Thus the integral of this gain times the speed droop over the interval of time, equal to the gate opening time, will completely open the wicket gates.

In the forthcoming analysis, all values are expressed in per unit. As is shown by Figure II-2-3, the position of the pilot valve is determined by the frequency deviation and droop signals.

Frequency deviation signal $B \Delta f$

Permanent speed droop signal $\delta'_p G$

Temporary speed droop signal; refer to Figure II-2-7 where the equivalent speed droop action is shown.

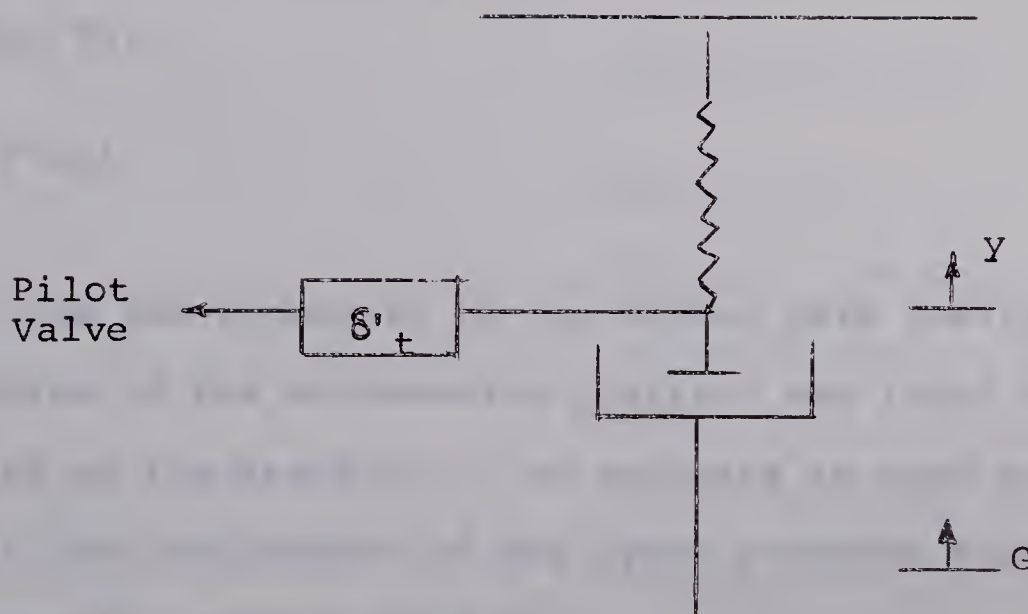


Figure II-2-7

$$Cs(G - y) = Dy$$

$$\frac{\delta'_t y}{s + D/C} = \frac{s \delta'_t G}{s + d} = \frac{s \delta'_t G}{s + d} \quad (\text{II-2-10})$$

where $d = 1/T_r$

The pilot valve position

$$X_p = B \left[\frac{\Delta_f - s \delta'_t G}{B(s + 1/T_r)} - \frac{\delta'_p G}{B} \right] \quad (\text{II-2-11})$$

The servo-motor valve position

$$X_v = \frac{K' X_p}{s} \quad (\text{II-2-12})$$

$$\text{Let } \frac{\delta'_t}{B} = \delta_t, \quad \frac{\delta'_p}{B} = \delta_p, \quad K'B = K$$

$$X_v = K \left[\frac{\Delta_f - s \delta_t G}{s + 1/T_r} - \delta_p G \right] \quad (\text{II-2-13})$$

The turbine wicket gate position is a non-linear function of the servo-motor valve position (refer to Figure II-2-3), (Block #5).

$$G = F(X_v) \quad (\text{II-2-14})$$

The non-linearity of the wicket gate position as a function of the servo-valve position was found to have little effect on the stability. An analysis is made on this effect after the development of the hydro transfer function (see Chapter II, Section II-3-13).

Thus the simulated equations representing the speed governing model, expressed in per unit are

$$P_t - P_d \Delta f - P_l = 2H \frac{df}{dt} \quad (\text{II-2-9})$$

$$X_v = K \left[\Delta f - \frac{s \delta_t X_v}{s + 1/T_r} - \delta_p X_v \right] \quad (\text{II-2-15})$$

Section II-2-3 Conclusion

The per unit torque equations may be replaced with per unit power equations and the rotating inertia power may be assumed constant considering speed deviations of less than 4%.

Section II-3 The Hydro Model

Section II-3-0 Introduction

The transfer function relating the turbine output power to the wicket gate position is determined in this section for systems with and without a surge tank.

$$\text{Turbine output power} \quad P_T = \frac{Q_t H_t \eta}{550} \quad (\text{II-3-1})$$

$$Q_t = Q(H, G, W_c, E_c) = G C_d \sqrt{2g H_t} \quad \text{Flow rate} \quad (\text{II-3-2})$$

$$H_t = H(Q_t, L_e) \quad \text{Piezometric head}$$

$$\eta \quad \text{Efficiency}$$

$$L_e \quad \text{Effective length of the water column as seen by the turbine}$$

$$G \quad \text{Wicket gate position}$$

$$W_c = W(H) \quad \text{Water compressibility}$$

$$E_c = E(H) \quad \text{Elasticity of the conduit}$$

$$C_d \quad \text{Coefficient of Discharge}$$

As may be seen by the above functional relationships, the determination of the turbine output power as a function of the gate position is a complex problem as it involves a large number of interrelated variables. Therefore, to obtain a solution, the problem is sectioned in the following manner.

A. Systems without a surge tank are considered first.

1. Idealizations are made to the problem in Sections II-3-1 and II-3-2.
2. The relationships between the variables are established in Sections II-3-3 to II-3-5.
3. Approximations are made to the equations in Section II-3-6.
4. The solution, to the approximated equations, is outlined and interpreted in Section II-3-7.
5. A transient hydraulic performance - transmission line analogy is made in Section II-3-8.
6. The transfer function is obtained for the fundamental component in the pressure and flow rate variations in Section II-3-9. The higher harmonic components are considered in Section II-3-12.

B. Systems with a surge tank are considered next.

1. Idealizations are made in Section II-3-10.
2. The transfer function is obtained in Section II-3-11.

The effect of the turbine wicket gate non-linearity and the higher harmonics in the pressure and flow rate variations are considered in Section II-3-13 and Section II-3-12 respectively.

Section II-3-1 The Idealization of the Conduit

The conduit from the reservoir to the turbine is assumed to lie in a straight line. If the conduit length remains the same, this assumption has no effect on the problem as seen by the turbine (refer to Figures II-3-1 and II-3-2).

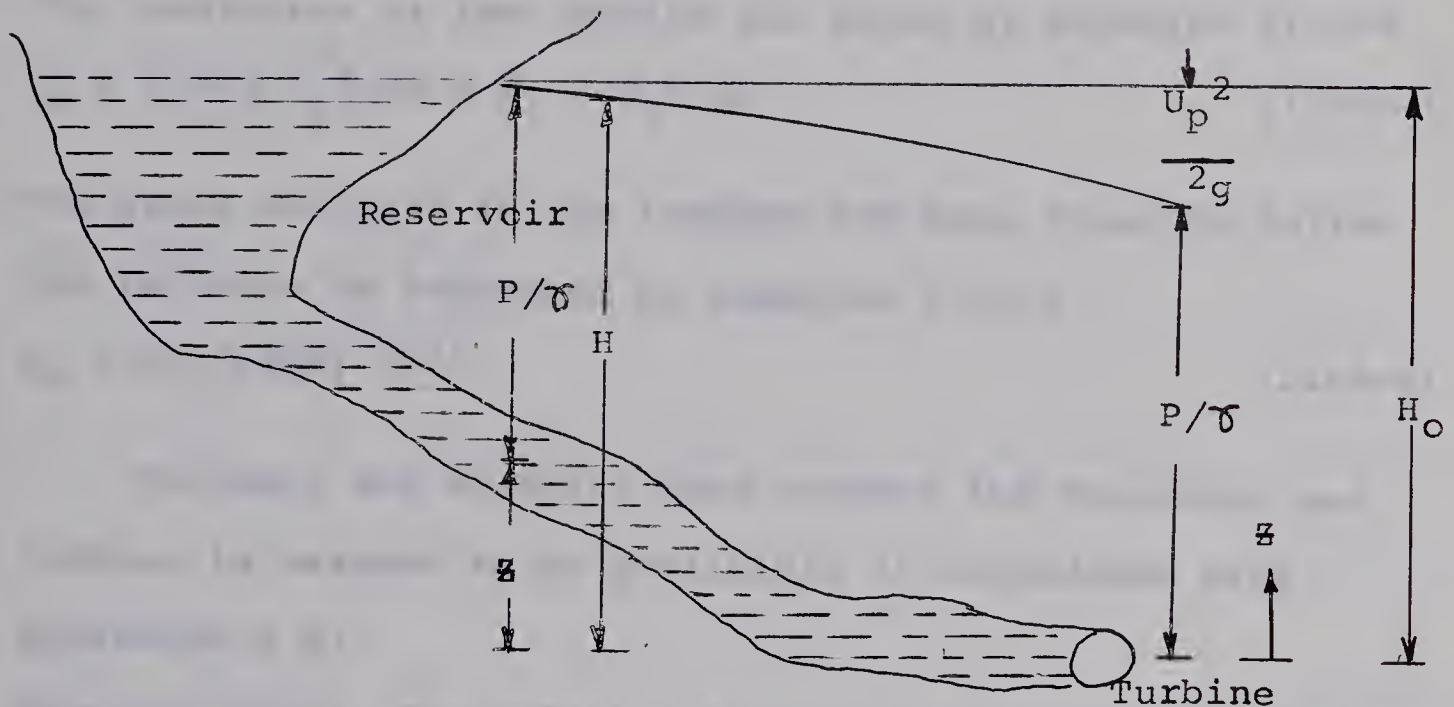


Figure II-3-1

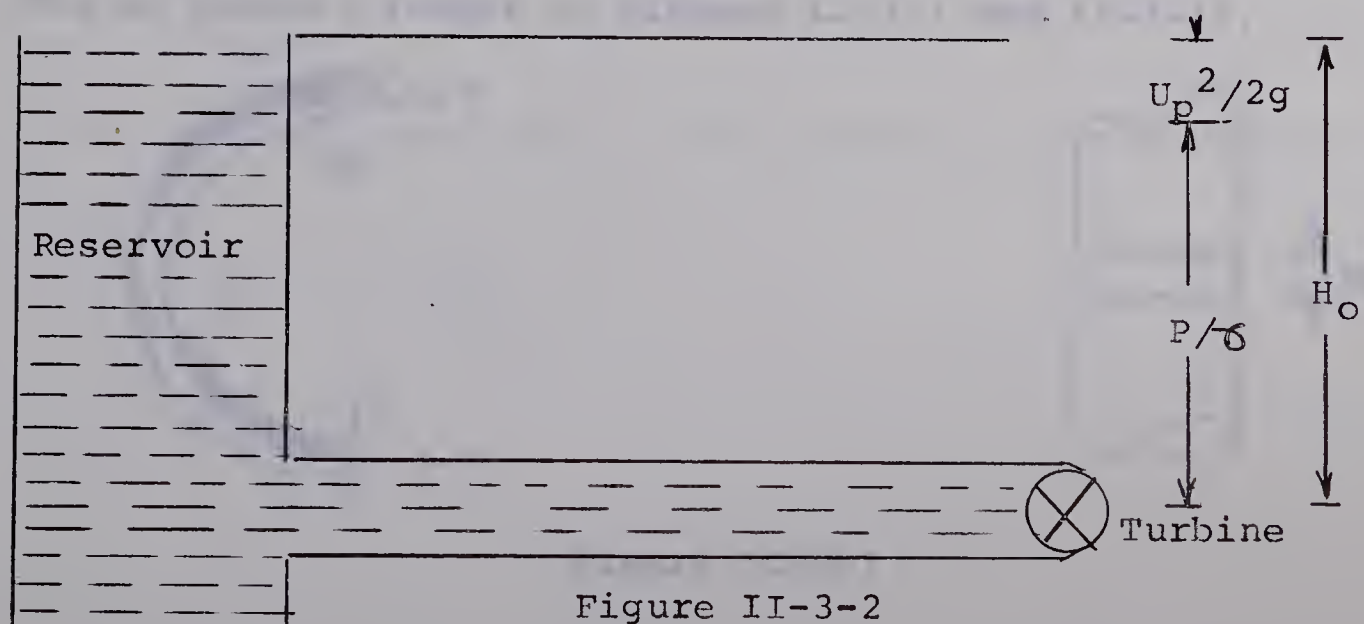


Figure II-3-2

Section II-3-2 The Idealization of the Piezometric Head

Assuming no friction, and steady state conditions, the pressure and flow rate in the conduit may be determined from Bernoulli's equation.

$$H_o = P/\gamma + Z + U_p^2/2g \quad (\text{II-3-3})$$

The conditions at the turbine are given by equation II-3-4

$$H_o = P/\gamma + U_p^2/2g = H_t + U_p^2/2g \quad (\text{II-3-4})$$

The water velocity in the turbine has been found to follow the relation as expressed by equation II-3-2.

$$Q_t = GC_d \sqrt{2gH_t} \quad (10) \quad (\text{II-3-2})$$

Further, the velocity head between the reservoir and turbine is assumed to be negligible in accordance with reference # 17.

Section II-3-3 The Relation Between the Pressure Variation, Change in Conduit Area and Fluid Compressibility

The change in area of the conduit as a function of the change in pressure may be determined by cutting a section out of conduit (refer to Figures II-3-3 and II-3-4).



Figure II-3-3

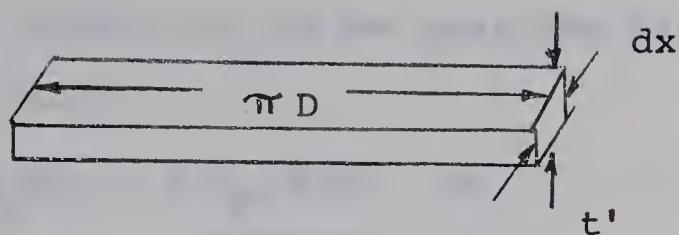


Figure II-3-4

$$\Delta T = \gamma \, dH \, R = \gamma \, dH \, D_p / 2 \quad (\text{II-3-5})$$

$$\Delta L = \frac{\pi D_p^2 \gamma \, dH}{2 \, t' E} \quad (\text{II-3-6})$$

$$\Delta D_p = \frac{\Delta L}{\pi} = \frac{D_p^2 \gamma \, dH}{2 \, t' E} \quad (\text{II-3-7})$$

$$\Delta A_p = \pi/4 \left[(D_p + \Delta D_p)^2 - D_p^2 \right] = \pi/4 \left[2\Delta D_p D_p + \Delta D_p^2 \right] \approx \pi/2 D_p \Delta D_p \quad (\text{II-3-8})$$

$$\Delta A_p = \frac{D_p^3 \gamma \, dH}{4 \, t' E} \quad (\text{II-3-9})$$

The change in volume over the element dx is given by equation II-3-10

$$dV_p = \frac{D_p^3 \gamma \, dH}{4 \, t' E} \, dx \quad (\text{II-3-10})$$

The change of the volume of the fluid is given by equation II-3-11.

$$K = -V \frac{dH}{dV} \gamma = -V \frac{dP}{dV_c} \quad (\text{II-3-11})$$

$$dV_c = -V \gamma \frac{dH}{K}$$

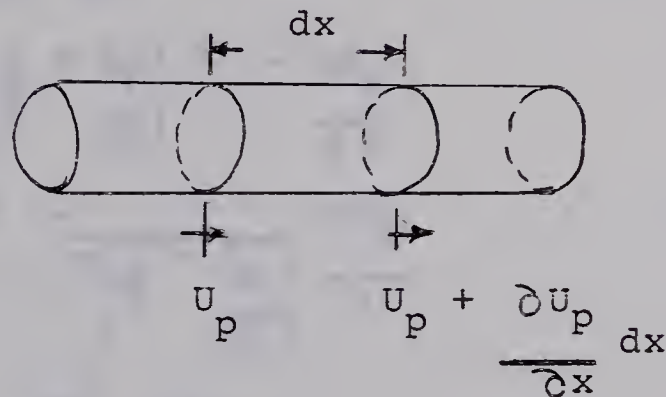
The total change in volume over the element dx is given by equation II-3-12.

$$dV_p + dV_c = \frac{V \delta dH}{K} + \frac{\pi D_p^3 \delta dH}{4 t' E} dx$$

$$dV_p + dV_c = (A_p dx) \delta dH \left[\frac{1}{K} + \frac{D_p}{t' E} \right] \quad (\text{II-3-12})$$

Section II-3-4 The Relation Between the Flow Rate Variation and the Change in Volume

The equation, relating the flow rate change to the volume change, may be determined by referring to Figure II-3-5.



$$\frac{dU_p}{dx} = \frac{\partial U_p}{\partial x} dx + \frac{\partial U_p}{\partial t} \bigg|_{x_2} dt - \frac{\partial U_p}{\partial t} \bigg|_{x_1} dt \quad (\text{II-3-13})$$

$$\text{Change in flow rate} = A_p dU_p \quad (\text{II-3-14})$$

The required change in volume due to the flow rate

$$\text{change} = A_p dU_p dt \quad (\text{II-3-15})$$

Assuming $\frac{\partial U_p}{\partial x} dt$ is constant over the element's length,

$$\text{the required change in volume} = \frac{\partial U_p}{\partial x} A_p dt dx \quad (\text{II-3-16})$$

As the change in flow rate is the resultant effect of the change in volume, one may equate equations II-3-12 and II-3-16.

$$A_p dx \frac{dH}{dt} \gamma \left[\frac{1}{K} + \frac{D_p}{tE} \right] = -A_p \frac{\partial U_p}{\partial x} dx dt$$

$$\frac{dH}{dt} \gamma \left[\frac{1}{K} + \frac{D_p}{tE} \right] = - \frac{\partial U_p}{\partial x} \quad (\text{II-3-17})$$

$$\frac{dH}{dt} = \frac{\partial H}{\partial t} + \frac{\partial H}{\partial x} \frac{dx}{dt} \quad (\text{II-3-18})$$

$$\gamma \left[\frac{\partial H}{\partial t} + U_p \frac{\partial H}{\partial x} \right] \left[\frac{1}{K} + \frac{D_p}{tE} \right] = - \frac{\partial U_p}{\partial x} \quad (\text{II-3-19})$$

$$\frac{\partial H}{\partial t} + U_p \frac{\partial H}{\partial x} = - \frac{1}{\gamma \left[\frac{1}{K} + \frac{D_p}{tE} \right]} \frac{\partial U_p}{\partial x}$$

$$\frac{\partial H}{\partial t} + U_p \frac{\partial H}{\partial x} = - \frac{b^2}{g} \frac{\partial U_p}{\partial x} \quad (\text{II-3-20})$$

Section II-3-5 The Relation Between the Pressure Variation and Water Inertia

The equation, relating the change in flow rate to the change in pressure, may be determined by referring to Figure II-3-6.

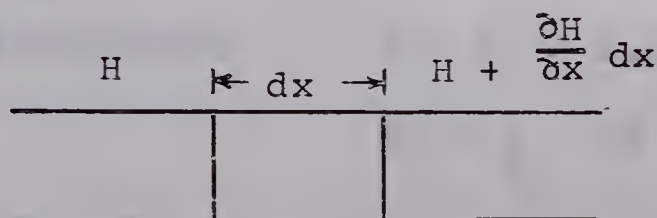


Figure II-3-6

$$F = \gamma \frac{dH}{dt} A_p = \gamma A_p \left[\frac{\partial H}{\partial x} dx + \frac{\partial H}{\partial t} \right]_{x_2} dt - \frac{\partial H}{\partial t} \Big|_{x_1} dt \quad (\text{II-3-21})$$

$$= -ma = -(\rho A_p dx) \frac{dU_p}{dt} \quad (\text{II-3-22})$$

$$\frac{dU_p}{dt} = \frac{\partial U_p}{\partial t} + \frac{\partial U_p}{\partial x} \frac{dx}{dt} = \frac{\partial U_p}{\partial t} + U_p \frac{\partial U_p}{\partial x} \quad (\text{II-3-23})$$

Assuming $\frac{\partial H}{\partial t} dt$ is constant over the elements length,

equation II-3-21 becomes equation II-3-24.

$$\gamma A_p \frac{\partial H}{\partial x} dx = -\rho A_p dx \left[\frac{\partial U_p}{\partial t} + U_p \frac{\partial U_p}{\partial x} \right]$$

$$g \frac{\partial H}{\partial x} = - \left[\frac{\partial U_p}{\partial t} + U_p \frac{\partial U_p}{\partial x} \right] \quad (\text{II-3-24})$$

Section II-3-6 The Approximation of the Equations

The two equations, relating the pressure and flow variations in the conduit under transient conditions, are as follows:

$$\frac{\partial H}{\partial t} + U_p \frac{\partial H}{\partial x} = -\frac{b^2}{g} \frac{\partial U_p}{\partial x} \quad (\text{II-3-20})$$

$$-g \frac{\partial H}{\partial x} = \frac{\partial U_p}{\partial t} + U_p \frac{\partial U_p}{\partial x} \quad (\text{II-3-24})$$

The boundary conditions of these equations, neglecting pipe discontinuities are:

At the top of the reservoir $H = 0, U = 0, A_r = \infty$

At the turbine $U_t = \frac{G}{A_t} C_d \sqrt{2gH} \quad (\text{II-3-2})$

At the reservoir conduit junction $H = H_0, U = 0, A_r = \infty$

The initial condition is $H = H_0$, due to the idealization of the problem made in Section II-3-2.

The problem may be simplified by approximating equations II-3-20 and II-3-24 and then obtaining two second order equations. The approximations and the justification of them are now shown.

$$\frac{-b^2}{g} \frac{\partial U_p}{\partial x} = \frac{\partial H}{\partial t} - \frac{U_p}{g} \left[\frac{\partial U_p}{\partial t} + U_p \frac{\partial U_p}{\partial x} \right]$$

$$\frac{\partial H}{\partial t} = \frac{\partial U_p}{\partial x} \frac{U_p^2 - b^2}{g} + \frac{U_p}{g} \frac{\partial U_p}{\partial t} \quad (\text{II-3-25})$$

$$\text{As } b^2 \gg U_p^2$$

$$\frac{\partial H}{\partial t} = \frac{-b^2}{g} \frac{\partial U_p}{\partial x} \quad (\text{II-3-26})$$

Substituting equation II-3-20 into equation II-3-24

$$-g \frac{\partial H}{\partial x} = \frac{\partial U_p}{\partial t} - \frac{U_p g}{b^2} \left[\frac{\partial H}{\partial t} + U_p \frac{\partial U_p}{\partial x} \right]$$

$$\frac{\partial H}{\partial x} \left[\frac{(2U_p^2 - b^2)}{b^2} \right] g = \frac{\partial U_p}{\partial t} \quad (\text{II-3-27})$$

$$\frac{\partial U_p}{\partial t} \approx -g \frac{\partial H}{\partial x} \quad (\text{II-3-28})$$

Two second order equations are now obtained by differentiating equations II-3-26 and II-3-28 and substituting.

Differentiating equation II-3-26

$$\frac{\partial^2 H}{\partial t^2} = \frac{-b^2}{g} \frac{\partial U_p}{\partial x \partial t} \quad (\text{II-3-29})$$

$$\frac{\partial^2 H}{\partial x \partial t} = \frac{-b^2}{g} \frac{\partial U_p}{\partial x^2} \quad (\text{II-3-30})$$

Differentiating equation II-3-28

$$\frac{\partial^2 U_p}{\partial t^2} = -g \frac{\partial^2 H}{\partial t \partial x} \quad (\text{II-3-31})$$

$$\frac{\partial^2 U_p}{\partial x \partial t} = -g \frac{\partial^2 H}{\partial x^2} \quad (\text{II-3-32})$$

From equations II-3-29 to II-3-32, the following expressions are derived.

$$b^2 \frac{\partial^2 H}{\partial x^2} = \frac{\partial^2 H}{\partial t^2} \quad (\text{II-3-33})$$

$$b^2 \frac{\partial^2 U_p}{\partial x^2} = \frac{\partial^2 U_p}{\partial t^2} \quad (\text{II-3-34})$$

The problem is now reduced to that of solving equations II-3-33 and II-3-34, subject to the boundary and initial conditions given at the beginning of the section.

Section II-3-7 The Solution and Interpretation of the Equations

The general solution of equations II-3-33 and II-3-34 is known to be (15):

$$H - H_0 = F(t + x/b) + f(t - x/b) \quad (\text{II-3-35})$$

$$U_p - U_{p0} = g/b [f(t - x/b) - F(t + x/b)]$$

$$Q_p - Q_{p0} = \frac{A_p g}{b} [f(t - x/b) - F(t + x/b)] \quad (\text{II-3-36})$$

In deriving the equations, x and U_p were measured positive from the reservoir. In the application of the solution, it is more convenient to measure x positive from the turbine as this is where the disturbance occurs.

Rewriting the equations on this basis

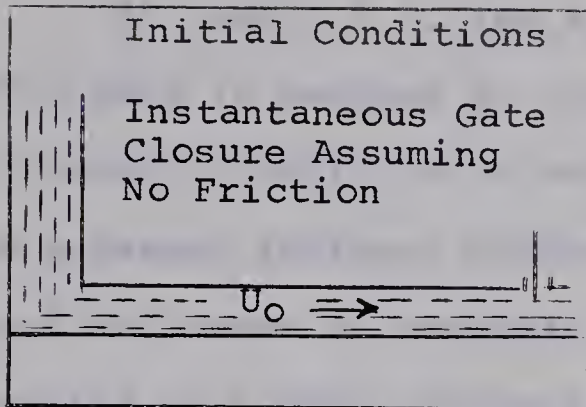
$$H - H_0 = F(t - x/b) + f(t + x/b) = F + f \quad (\text{II-3-37})$$

$$Q_p - Q_{p0} = \frac{A_p g}{b} [F(t - x/b) - f(t + x/b)] = -\frac{A_p g}{b} [F - f]$$

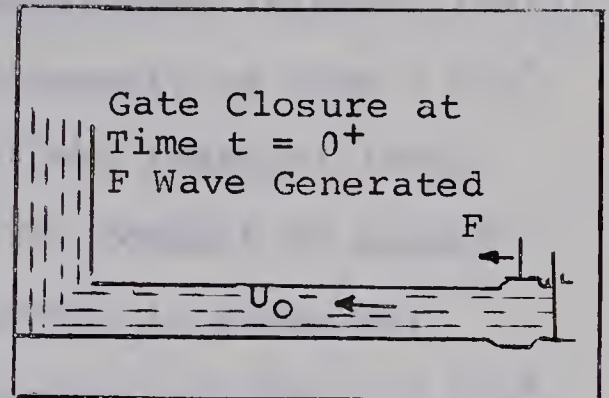
Although equations II-3-37 and II-3-38, do not satisfy the boundary conditions, they are still useful as they enable a physical picture of the hydraulic transient performance to be obtained. When the hydraulic performance is understood, it is possible to use a graphical method to solve the equations subject to the boundary conditions.

To gain an insight into the hydraulic transient performance, the interpretation of the equations and their graphical solution is outlined for rapid gate closures. Following this, the graphical solution is outlined for slow and oscillatory gate movements.

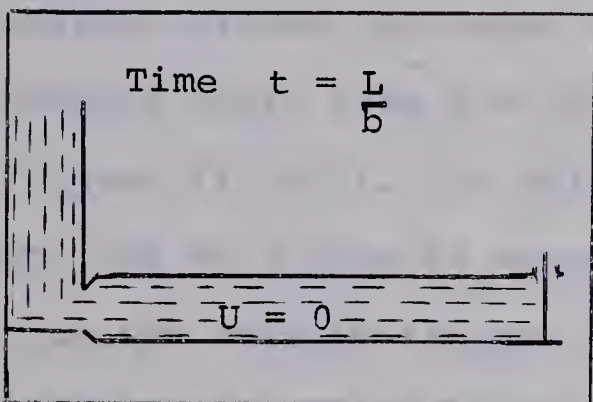
Physical interpretation of the equations for rapid gate closures is shown in Figures II-3-7 to II-3-12.



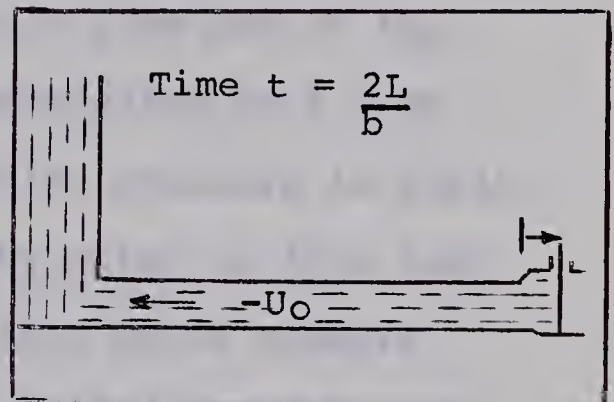
II-3-7



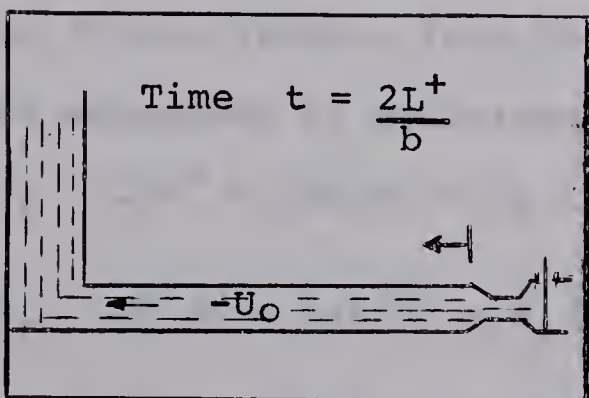
II-3-8



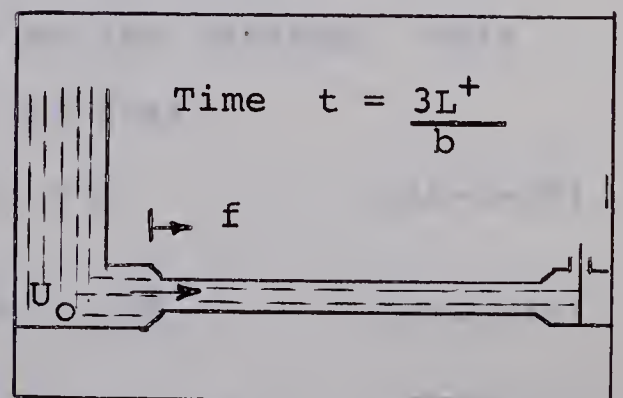
II-3-9



II-3-10



II-3-11



II-3-12

At time $t = \frac{4L^+}{a}$, the process starts over again.

At time $t = 0$, the system is undisturbed (Figure II-3-7). The gate is assumed to close instantaneously at time $t = 0^+$ (Figure II-3-8). As a consequence of the inertial force, a pressure increase ensues, causing the conduit to expand and the water to compress. This increase in pressure, called an F wave, arrives at the reservoir at time $t = L/b^+$, and stops the flow into the pipe. Due to the increased pressure in the conduit, water flows out of the conduit until time $t = 2L/b^+$ (Figure II-3-10). The force of the inertia allows the water to continue to flow out of the conduit until time $t = 3L/b^+$, thus generating an F wave (Figure II-3-11). At this time, suction pressure is built up, and an f wave is generated causing water to flow back into the conduit (Figure II-3-12). This cycle repeats itself and results in an oscillation called water-hammer.

During the time intervals, $(0^+ + 2nL/b) \leq t \leq (L/b^+ + 2nL/b)$, an F wave travels from the turbine to the reservoir. During the time intervals, $(L/b^+ + 2nL/b) \leq t \leq (2L/b^+ + 2nL/b)$, an f wave travels from the reservoir to the turbine. This is expressed in equations II-3-39 to II-3-44.

$$H_t (L/b^+ + 2nL/b) - H_r (0^+ + 2nL/b) = F \quad (\text{II-3-39})$$

$$H_r (2L/b^+ + 2nL/b) - H_t (L/b^+ + 2nL/b) = f \quad (\text{II-3-40})$$

$$Q_t (L/b^+ + 2nL/b) - Q_r (0^+ + 2nL/b) = \frac{-A_p g}{b} F \quad (\text{II-3-41})$$

$$Q_r (2L/b^+ + 2nL/b) - Q_t (L/b^+ + 2nL/b) = \frac{-A_p g}{b} f \quad (\text{II-3-42})$$

$$\begin{aligned}
& H_t (L/b^+ + 2nL/b) - H_r(0^+ + 2nL/b) \\
&= \frac{-b}{A_p g} [Q_t(L/b^+ + 2nL/b) - Q_r(0^+ + 2nL/b)] \quad (II-3-43)
\end{aligned}$$

$$\begin{aligned}
& H_r (2L/b^+ + 2nL/b) - H_t(L/b^+ + 2nL/b) \\
&= \frac{-b}{A_p g} [Q_r(2L/b^+ + 2nL/b) - Q_t (L/b^+ + 2nL/b)] \quad (II-3-44)
\end{aligned}$$

As the conditions are the same: between L/b^+ and $2L/b$, $2L/b^+$ to $3L/b$, equations II-3-43 and II-3-44 are rewritten as II-3-45 and II-3-46.

$$H_t 2(n+1) L/b - H_r(2n+1) L/b = \frac{-b}{A_p g} [Q_t 2(n+1) L/b - Q_r(2n+1) L/b] \quad (II-3-45)$$

$$H_r(2n+1) L/b - H_t(2n) L/b = \frac{-b}{A_p g} [Q_r(2n+1) L/b - Q_t(2n) L/b] \quad (II-3-46)$$

The graphical solution of the equations II-3-45 and II-3-46, subject to the boundary condition $Q_t = C_d G \sqrt{2gH}$, is shown in Figure II-3-13. As the gate is assumed to close instantaneously, the flow $Q_t = 0$.

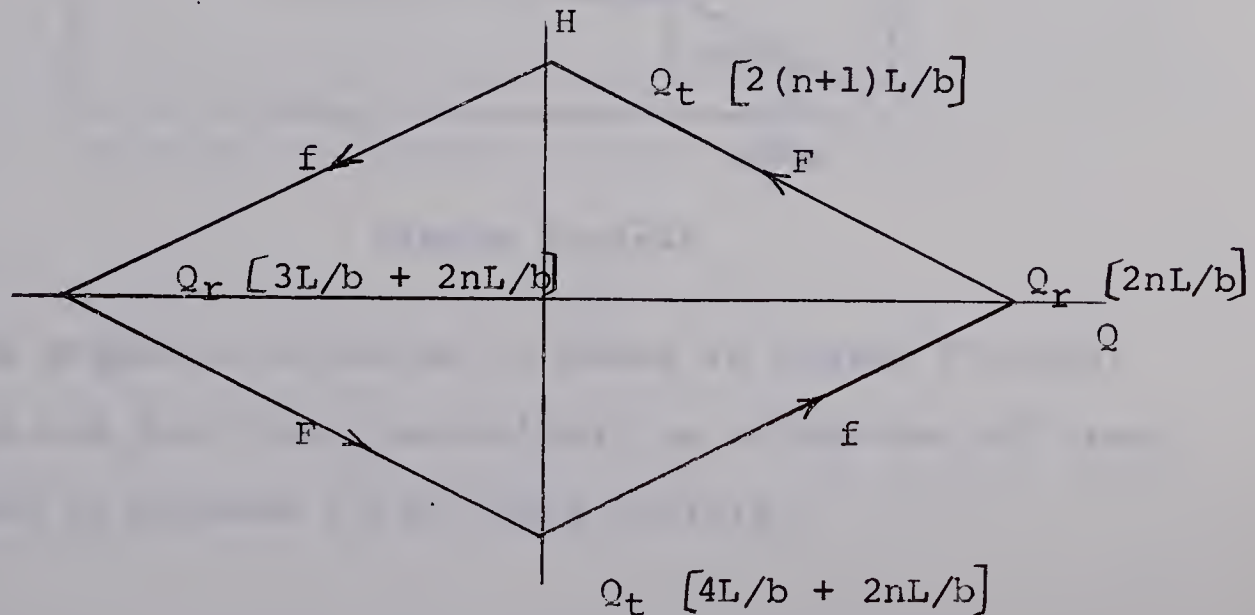


Figure II-3-13

The resulting pressure head oscillation at the turbine is shown in Figure II-3-14.

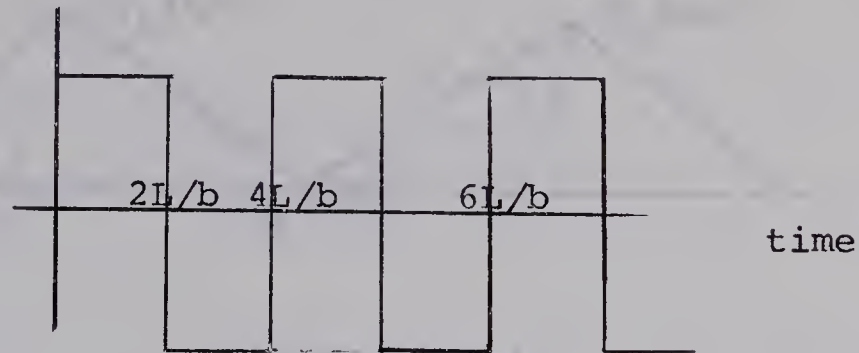


Figure II-3-14

The graphical solution of the equations for slow gate closures. The gate closure time is divided into $n(2L/b)$ intervals and a step approximation to the closure is made (refer to Figure II-3-15)

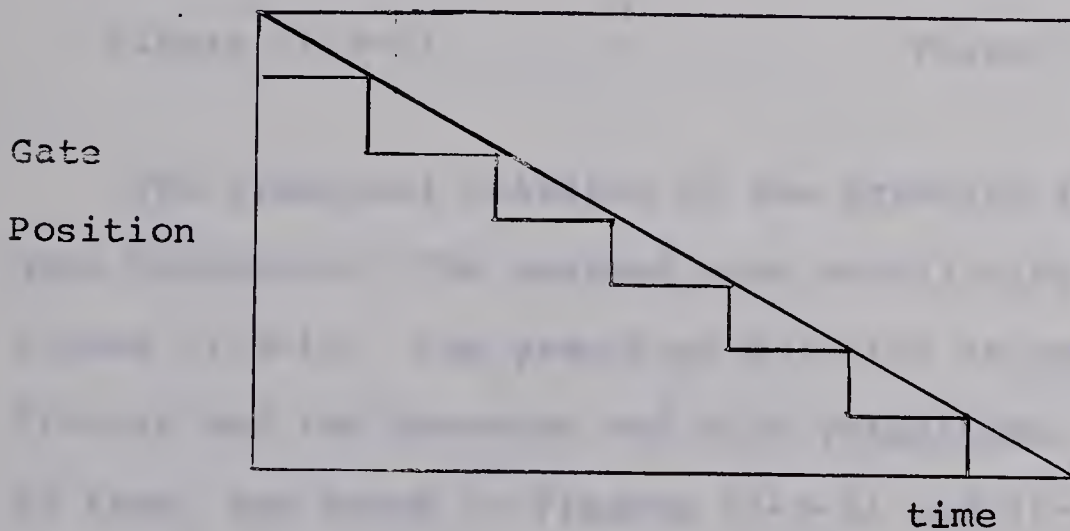


Figure II-3-15

The graphical solution is shown in Figure II-3-16. The head and flow rate variations, as a function of time, are shown in Figures II-3-17 and II-3-18.

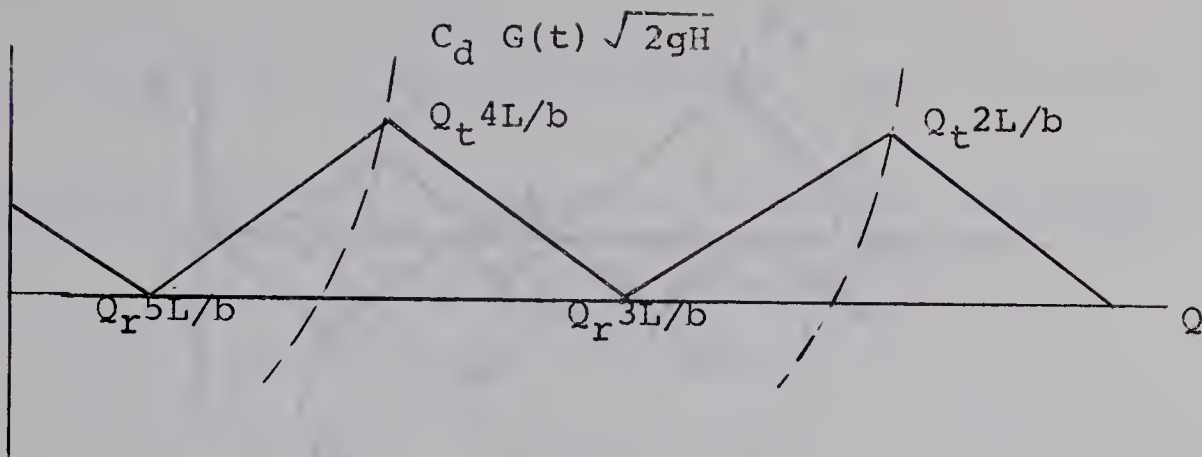


Figure II-3-16

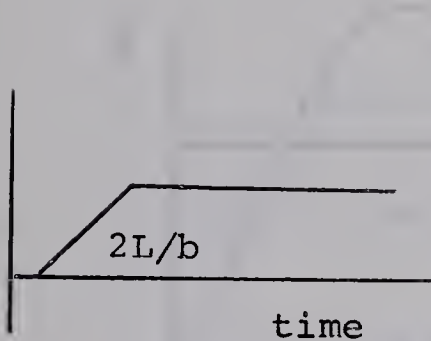


Figure II-3-17

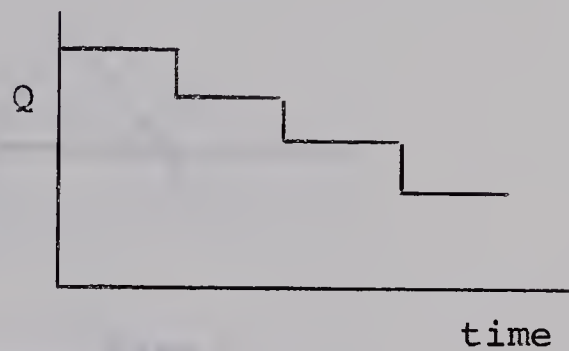


Figure II-3-18

The graphical solution of the equation for oscillatory gate movements. The assumed gate oscillation is shown in Figure II-3-19. The graphical solution is shown in Figure II-3-20 and the pressure and flow variations, as a function of time, are shown in Figures II-3-21 and II-3-22.

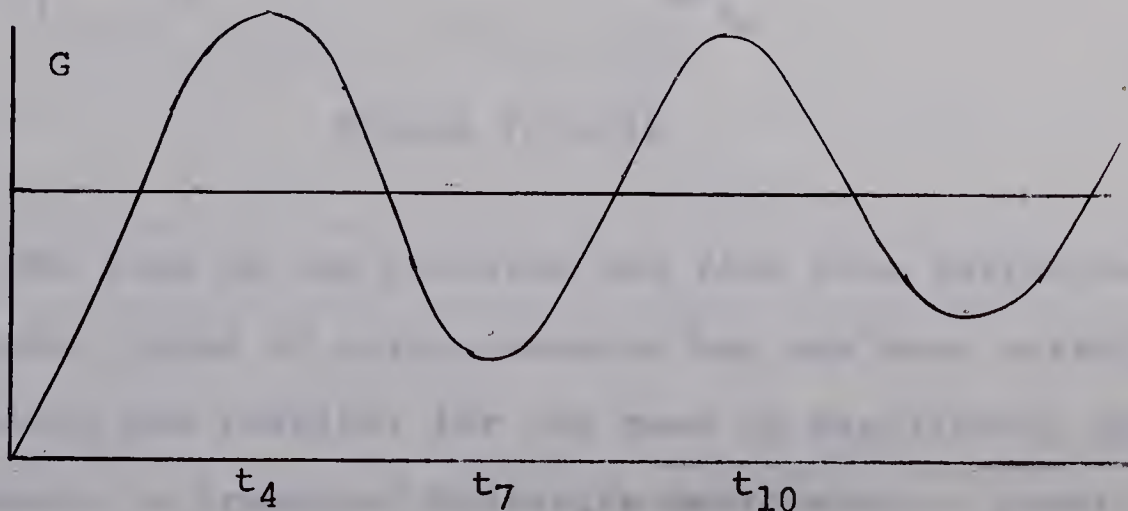


Figure II-3-19

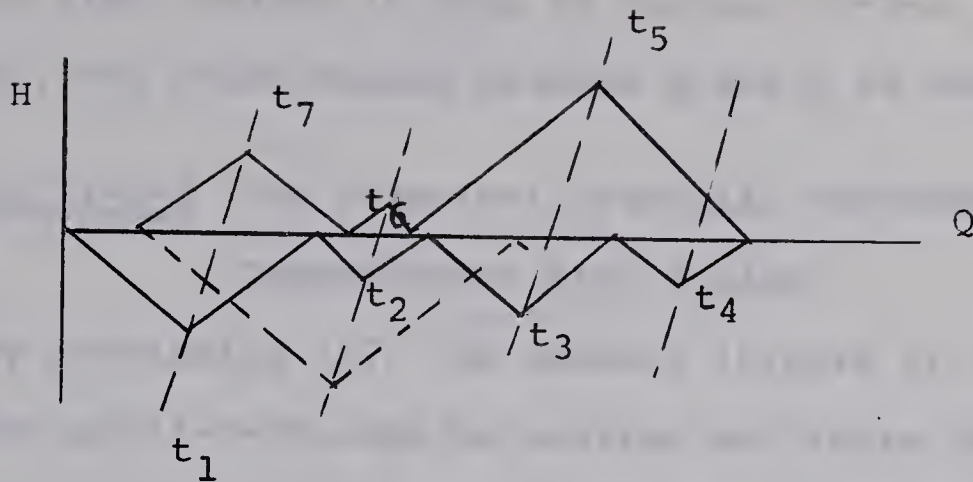


Figure II-3-20

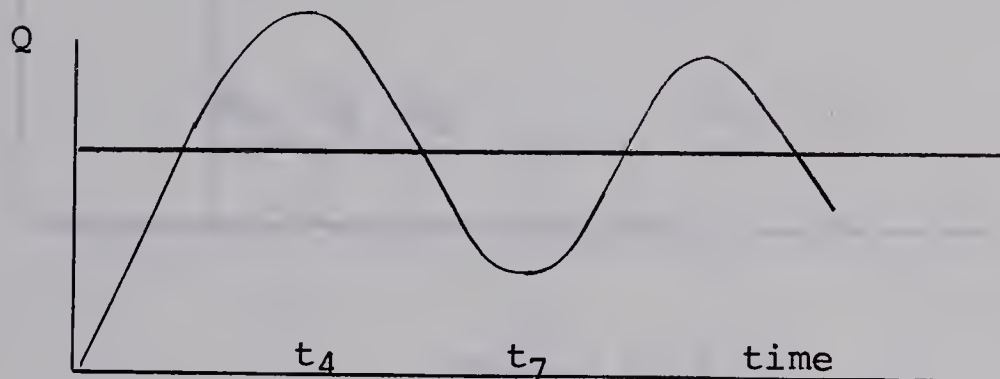


Figure II-3-21

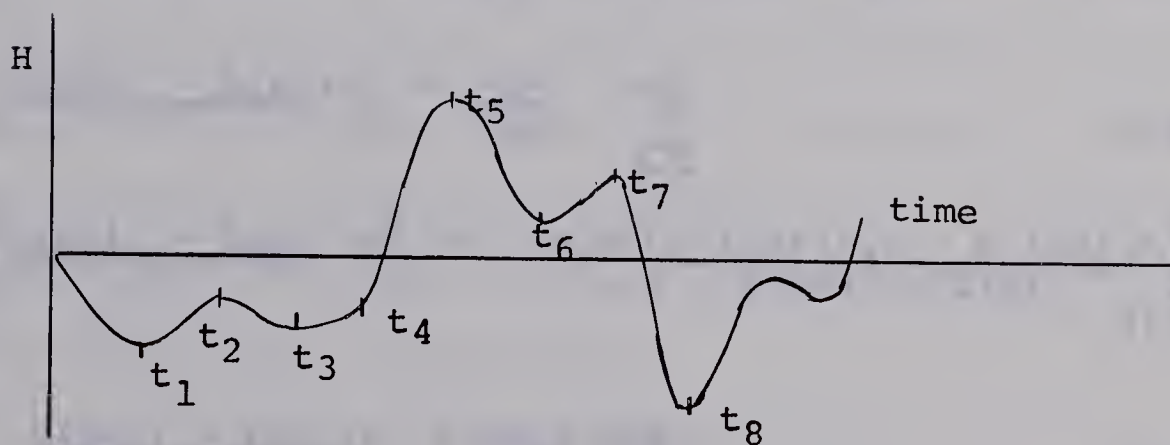


Figure II-3-22

The form of the pressure and flow rate variations for different types of gate movements has now been established. Utilizing the results, for the case of oscillatory gate movements, a transient hydraulic performance - power trans-

mission line analogy is made in Section II-3-8. From this analogy, the relationship between Q and H is established.

Section II-3-8 The Transient Hydraulic Performance -
Transmission Line Analogy

By sectioning (17) the conduit (Figure II-3-23), equations II-3-26 and II-3-28, may be written as finite different equations.

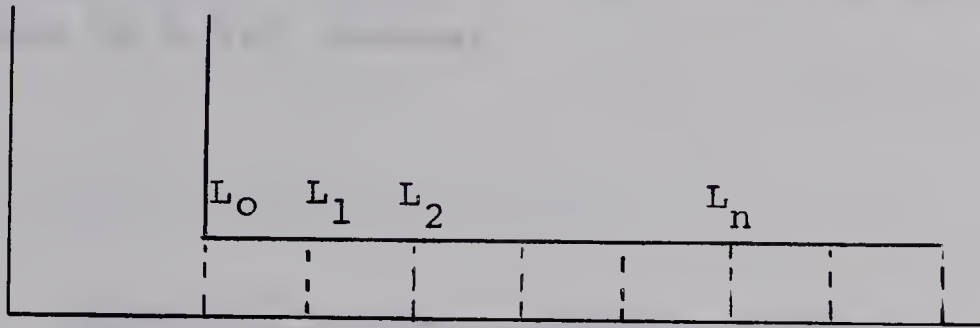


Figure II-3-23

$$\frac{dH}{dt} \left[\frac{L(n+1) + L(n)}{2}, t \right] = \frac{-b^2}{A_p g} \left[\frac{Q_{L(n+1)} - Q_{L(n)}}{L(n+1) - L(n)}, t \right] \quad (\text{II-3-47})$$

$$\frac{dH}{dt} \left[\frac{L(n+1) + L(n)}{2}, t \right] = \frac{-b^2}{A_p g} \frac{\Delta Q_p}{\Delta L} \quad (\text{II-3-48})$$

$$\frac{dQ_p}{dt} \left[\frac{L(n+1) + L(n)}{2}, t \right] = -A_p g \left[\frac{H_{L(n+1)} - H_{L(n)}}{L(n+1) - L(n)}, t \right] \quad (\text{II-3-49})$$

$$\frac{dQ_p}{dt} \frac{L(n+1) + L(n)}{2}, t = -A_p g \frac{\Delta H}{\Delta L} \quad (\text{II-3-50})$$

The pressure and flow variations, over the Section ΔL , are lumped and represented as capacitance and inductance respectively (refer to II-3-24).

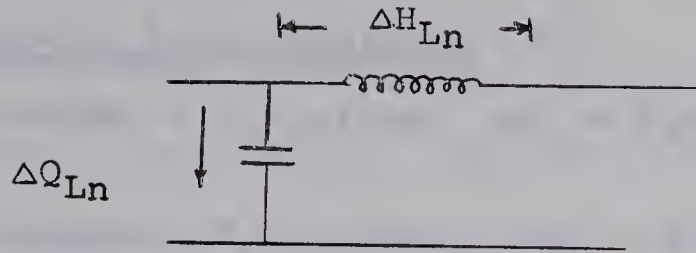


Figure II-3-24

The turbine sees the circuit shown by Figure II-3-25, as the reservoir offers zero impedance to the variations and appears as a D.C. source.

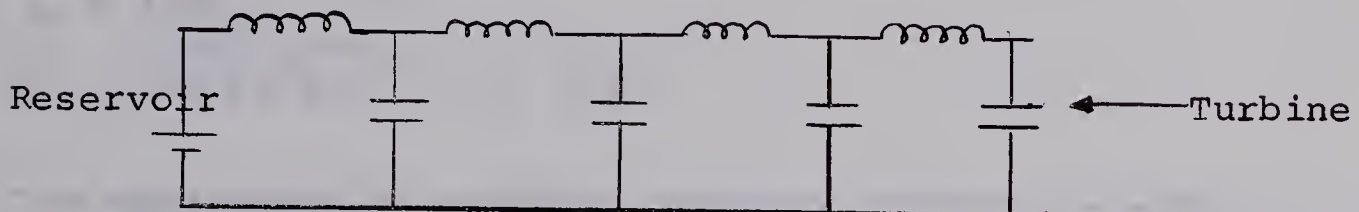


Figure II-3-25

To make the transient hydraulic performance analogous to the power transmission line, it is necessary to have an undamped source equivalent to voltage. This is attained by considering sustained pressure oscillations resulting from undamped gate oscillations.

Equivalence

Hydraulic System

H (head)
 $-\Delta Q$ (flow rate)
 $1/A_p g$
 $A_p g/b^2$
 W

Power Transmission Line

E (voltage)
 I (current)
 L' (inductance)
 C (capacitance)
 ξ (frequency)

Transmission Line Equations (16)

$$E_s = E_r \cosh mL + I_r Z_o \sinh mL \quad \Delta H_t = H_r \cosh mL + \Delta Q_r Z_o \sinh mL \quad (\text{II-3-51})$$

$$I_s = I_r \cosh mL + E_r Y_o \sinh mL - \Delta Q_t = Q_r \cosh mL + H_r Y_o \sinh mL \quad (\text{II-3-52})$$

where

E_s	sending voltage	ΔH_t	turbine pressure deviation
I_s	sending current	ΔQ_t	turbine flow rate deviation
E_r	receiving voltage	ΔH_r	reservoir pressure deviation
I_r	receiving current	ΔQ_r	reservoir flow rate deviation

$$Z_o = \sqrt{\frac{j \xi L'}{j \xi C}} = \sqrt{L'/C} = b/A_p g$$

$$Y_o = 1/Z_o$$

$$m = \sqrt{(j \xi)^2 L' C} = j \xi / b$$

The equivalent π circuit (refer to Figure II-3-26)

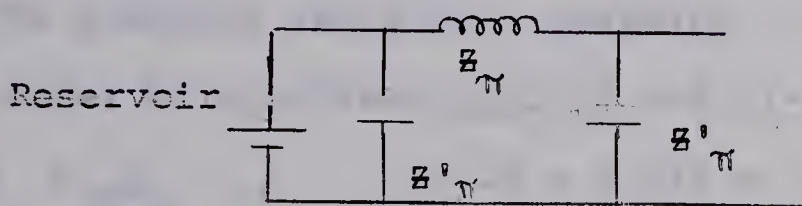


Figure II-3-26

$$Z_\pi = Z_o \sinh mL$$

$$Z_\pi = \frac{b}{A_p g} j \left[\left(\frac{\xi L}{b} \right) - \frac{\left(\frac{\xi L}{b} \right)^3}{3!} + \frac{\left(\frac{\xi L}{b} \right)^5}{5!} - \frac{\left(\frac{\xi L}{b} \right)^7}{7!} + \dots \right] \quad (\text{II-3-53})$$

$$Z'_\pi = \frac{Z_o \sinh mL}{\cosh mL - 1}$$

$$Z''_\pi = \frac{b}{A_p g} j \left[\frac{1 - \frac{(L/b)^2}{3!} + \frac{(L/b)^4}{5!} + \dots}{-\frac{(L/b)}{2!} + \frac{(L/b)}{3!} - \frac{(L/b)^5}{6!} + \dots} \right] \quad (\text{II-3-54})$$

The natural frequency of the governing loop (ξ), is normally less than 0.7 radians per second. Assuming L/b to be less than 0.6, the impedances offered to the fundamental components of the pressure variation are given in equations II-3-55 and II-3-56.

$$Z_{\pi} = \frac{b}{A_p g} j (0.42 - 0.0122 + 0.0001 \text{ -----} + \text{-----})$$

$$Z_{\pi} \approx \frac{j \xi L}{A_p g} \quad (\text{II-3-55})$$

$$Z'_{\pi} = \frac{j b}{A_p g} \left[\frac{1 - 0.0294 + 0.00026}{-0.21 + 0.0039} \right] \quad (\text{II-3-56})$$

$$Z'_{\pi} = \frac{2b^2}{LA_p g} \frac{1}{j \xi}$$

The impedances offered to the third harmonic component of the pressure variation, assuming $\xi = 0.7$ and $L/b = 0.6$, are given by equations II-3-57 and II-3-58.

$$Z_{\pi 3} = \frac{b}{A_p g} j (1.26 - 0.333 + .0263 + \text{-----})$$

$$Z_{\pi 3} = j 0.927 \frac{b}{A_p g} = 1.22 j (3 \xi) \frac{L}{A_p g} \quad (\text{II-3-57})$$

$$Z'_{\pi 3} = \frac{b}{A_p g} \left[\frac{1 - 0.264 + .0209}{-0.63 + .083} \text{-----} \right]$$

$$Z'_{\pi 3} = \frac{1.37}{j} \frac{b}{A_p g} = \frac{2.18}{j 3 \xi} L \frac{2 b^2}{A_p g} \quad (\text{II-3-58})$$

As the reservoir offers zero impedance to the A.C. variations, the capacitor at the reservoir is shorted out. Thus, the relationship between the fundamental component of the pressure variation and the flow rate variation is

given by equation II-3-59.

$$\Delta H_t = (-\Delta Q_t) \frac{B_{\pi} B_{\pi}^0}{B_{\pi} + B_{\pi}^0} = (-\Delta Q_t) \frac{B_{\pi}}{\frac{B_{\pi}^0}{B_{\pi}} + 1}$$

$$\Delta H_t = (-\Delta Q_t) \frac{j \xi L / A_p g}{1 + \frac{(j \xi)^2}{2} \left(\frac{L}{b}\right)^2}$$

$$\Delta H_t \approx (-\Delta Q_t) \frac{j \xi L}{A_p g}, \text{ as } \frac{(j \xi)^2}{2} \left(\frac{L}{b}\right)^2 \leq .028 \quad (\text{II-3-59})$$

Expressing equation II-3-59 in operator notation, one obtains equation II-3-60.

$$\Delta H_t \approx s \left(\frac{L}{A_p g} \right) (-\Delta Q_t) \quad (\text{II-3-60})$$

The relationship between the third harmonic component of the pressure variation and the flow rate variation is given by equation II-3-61.

$$\Delta H_{t3} = \frac{B_{\pi 3} B_{\pi 3}^0}{B_{\pi 3} + B_{\pi 3}^0} (-\Delta Q_{t3})$$

$$\Delta H_{t3} = \frac{j 0.927 \frac{b}{A_p g} (-\Delta Q_{t3})}{1 - 0.677}$$

$$\Delta H_{t3} = (-\Delta Q_{t3}) \left[3.4 j (3 \xi) \frac{L}{A_p g} \right] \quad (\text{II-3-61})$$

The impedances offered to the fundamental and third harmonic components of the pressure variations are considerably different as may be seen by equations II-3-59 and II-3-61. Equation II-3-59 shows that storage elements do

not effect the relationship between ΔQ and ΔH for the fundamental frequency component. Equation II-3-61 shows that the inertia and storage elements act as a tuned circuit to the third harmonic frequency.

The significance of the water storage elements and the non-linearity of the π circuit impedance depends on the magnitude of the harmonic components in the pressure variations. Thus, it is pertinent to determine the source and magnitude of the harmonics, as there is no point in attempting to represent the above factors if the harmonic components are not present in the simulation.

As the relationship between ΔH and ΔQ in the conduit is linear, the harmonics are created by the non-linear boundary condition (equation II-3-2). If equation II-3-2 can be linearized for small gate movements, the harmonic generator is removed, and equation II-3-60 yields a correct model of the hydraulic performance.

The boundary condition is linearized and the hydro transfer function, based on a rigid conduit and an incompressible fluid, is developed in Section II-3-9. Further considerations of the third harmonic component are made in Section II-3-12.

Section II-3-9 The Hydro Equation for Systems Without a Surge Tank

The relation between pressure and flow rate in the turbine is given by equation II-3-2.

$$Q_t = C_d G \sqrt{2gH_t}$$

$$dQ_t = C_d dG \sqrt{2gH_t} + C_d G \sqrt{2gH_t} \frac{dH_t}{2\sqrt{H_t}} \quad (\text{II-3-2})$$

$$\Delta Q_t = C_d \Delta G \sqrt{2gH_t} + C_d G \sqrt{2gH_t} \frac{\Delta H_t}{2\sqrt{H_t}} \quad (\text{II-3-62})$$

Placing equation II-3-62 on a per unit basis

$$\frac{\Delta Q_t}{Q_o} = \frac{\Delta G}{G_o} \sqrt{\frac{H_t}{H_o}} + \frac{1}{2} \frac{G}{G_o} \sqrt{\frac{H_o}{H_t}} \left(\frac{\Delta H_t}{H_o} \right) \quad (\text{II-3-63})$$

Equation II-3-63 has time varying coefficients. If the coefficients $\left(\sqrt{\frac{H_t}{H_o}} \right.$ and $\left. \frac{G}{G_o} \sqrt{\frac{H_o}{H_t}} \right)$ can be assumed constant

over a given range, the equation is linearized over this range. This may be accomplished by assuming small gate movements. (17)

The change in pressure for large gate movements was investigated by the author. It was found that the head deviation did not exceed 30% for a system with 2,000 ft. of conduit. Thus, for small gate movements, it is justified to assume head deviations of less than 10% H_o . This means that $\sqrt{H_t/H_o}$ does not vary more than plus or minus 5%.

Therefore equation II-3-63 may be assumed linear for small gate deviations as the coefficients $\sqrt{H_t/H_o}$ and $(G/G_o) \times (\sqrt{H_o/H_t})$ are almost constant.

As equation II-3-2 is linearized, equation II-3-60 is assumed to describe the hydraulic performance for the reasons stated in Section II-3-8. Placing equation II-3-60 on a per unit basis

$$\frac{\Delta H_t}{H_o} = - \frac{Q_o L}{H_o A_p g} s \frac{\Delta Q_t}{Q_o}$$

$$\frac{\Delta H_t}{H_o} = -T_w s \frac{\Delta Q_t}{Q_o} \quad (\text{II-3-64})$$

Substituting equations II-3-63 and II-3-64

$$\frac{\Delta Q_t}{Q_o} \left[T_w s + 2 \frac{G_o}{G} \sqrt{\frac{H_t}{H_o}} \right] = 2 \frac{G_o H_t}{G H_o} \frac{\Delta G}{G_o}$$

$$\frac{\Delta Q_t}{Q_o} = \frac{2 \frac{G_o H_t}{G H_o}}{T_w s + 2 \frac{G_o}{G} \sqrt{\frac{H_t}{H_o}}} \frac{\Delta G}{G_o} \quad (\text{II-3-65})$$

$$\frac{\Delta H_t}{H_o} = \frac{-2 \frac{G_o H_t}{G H_o} T_w s}{T_w s + 2 \frac{G_o}{G} \sqrt{\frac{H_t}{H_o}}} \frac{\Delta G}{G_o} \quad (\text{II-3-66})$$

The equation relating the turbine output power to the head and flow rate variations.

$$P_t = \frac{\gamma Q_t H_t \eta}{550} \quad (\text{II-3-1})$$

$$dP_t = \left[dH_t Q_t + H_t dQ_t \right] \frac{\gamma}{550} \eta \quad (\text{II-3-67})$$

Expressing equation II-3-67 in per unit

$$\frac{\Delta P_t}{P_o} = \frac{\Delta H_t Q_t}{H_o Q_o} + \frac{H_t \Delta Q_t}{H_o Q_o} \quad (\text{II-3-68})$$

Substituting equations II-3-65 and II-3-66 into equation II-3-68

$$\frac{\Delta P_t}{P_o} = \frac{\left[\frac{H_t}{H_o} \right]^{3/2} \left[1 - \frac{G}{G_o} \sqrt{\frac{H_o}{H_t}} T_w s \right]}{\frac{G}{G_o} \sqrt{\frac{H_o}{H_t}} \frac{T_w}{2} s + 1} \frac{\Delta G}{G_o} \quad (\text{II-3-69})$$

Equation II-3-69 has now been obtained in the Laplace domain. It will now be expressed in the time domain, by taking the Inverse Laplace transform, to consider the effects of the coefficients assuming a step change in gate position.

$$\frac{\Delta P_t(t)}{P_o} = \mathcal{L}^{-1} \left[\frac{2 \left[\frac{H_t}{H_o} \right]^2 \frac{G_o}{G T_w}}{s + 2 \frac{G_o}{G} \sqrt{\frac{H_t}{H_o}} \frac{1}{T_w}} - \frac{2 \left[\frac{H_t}{H_o} \right]^{3/2}}{s + 2 \frac{G_o}{G} \sqrt{\frac{H_t}{H_o}} \frac{1}{T_w}} \right] \frac{\Delta G}{G_o}$$

$$\frac{\Delta P_t}{P_o} = \left(\frac{H_t}{H_o} \right)^{3/2} \left[1 - 3 e^{-2 \frac{G_o}{G} \frac{H_t}{H_o} \frac{T}{T_w} t} \right] \frac{\Delta G}{G_o} \quad (\text{II-3-70})$$

The following deductions are made from equation II-3-70. The power output first decreases and then exponentially rises to its steady state value. The decrease in the pressure head tends to de-stabilize the system as it acts to decrease the power output and increases the exponential decay time. The term G_o/G acts to stabilize the system as it decreases the exponential decay time.

According to the rigid conduit and incompressible fluid theory, as may be realized from equation II-3-66, the smaller the initial gate position the larger is the head deviation for a given change in gate position. This effect becomes of greater significance as the length of the water column increases (T_w increases). However as the length of the water column increases, the storage capacity of the conduit also increases and as a result the pressure variation at the turbine is less severe. On this basis, coupled with the fact that the recovery time and the magnitude of the head deviations are inversely related for a given gate position, it is assumed that the resultant contribution of the terms G/G_o and H_t/H_o may be neglected. Thus equation II-3-69 is simplified to equation II-3-71.

$$\frac{\Delta P_t}{P_o} = \frac{2 (1 - T_w s)}{2 + T_w s} \frac{\Delta G}{G_o} \quad (\text{II-3-71})$$

$$\frac{\Delta P_t}{P_0} = \frac{2(1 - T_{ws})}{2 + T_{ws}} \frac{\Delta G}{G_0} + \text{Initial Conditions} \quad (\text{II-3-72})$$

Section II-3-10 The Idealization of the Surge Tank

The idealization of the physical system is portrayed in Figure II-3-27.

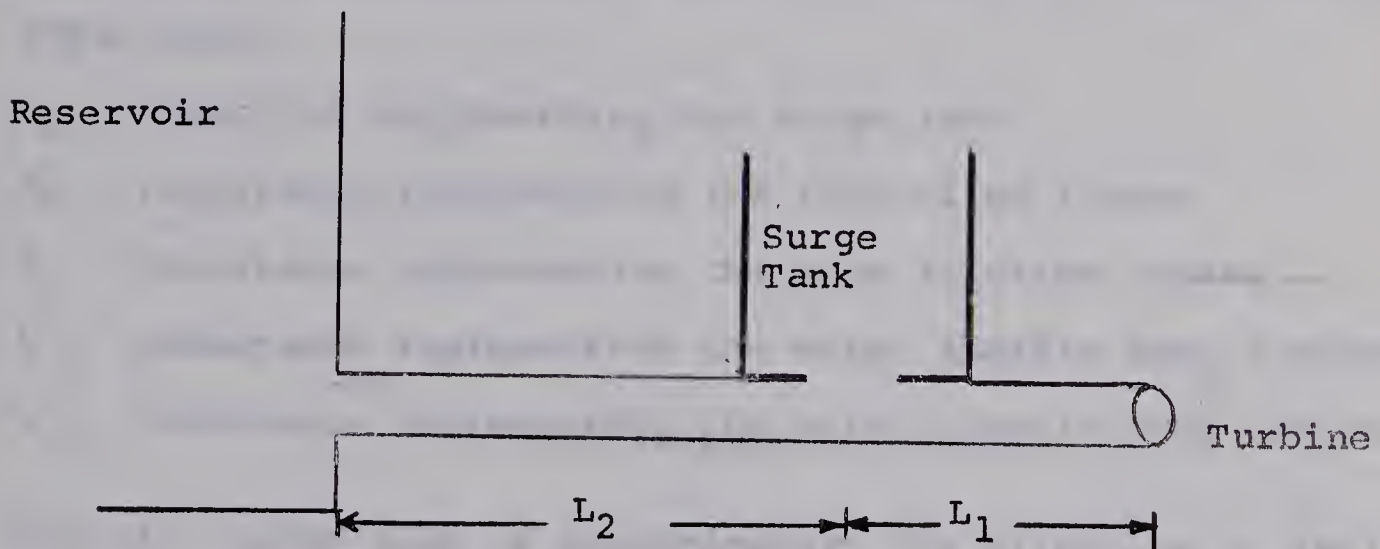


Figure II-3-27

The equivalent circuit as seen from the turbine is shown in Figure II-3-28.

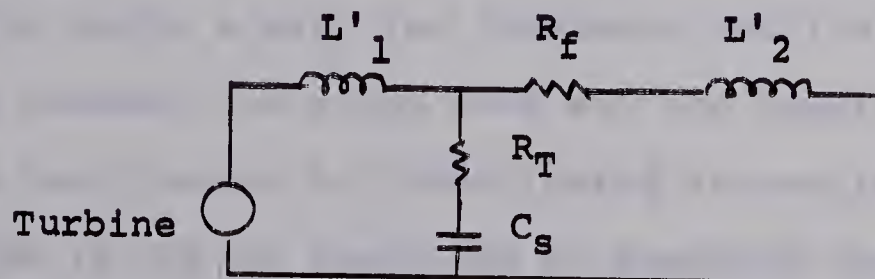


Figure II-3-28

The surge tank supplies or absorbs water as required by the flow rate variations and is represented as a capacitor. The pipe friction and throttling losses dissipate energy and are represented by resistances. These losses are significant as they are the only factor acting to damp out oscillations existing in the section of conduit between the reservoir and surge tank.

C_s capacitor representing the surge tank

R_t resistance representing the throttling losses

R_f resistance representing the pipe friction losses

L'_1 inductance representing the water inertia over distance L_1

L'_2 inductance representing the water inertia over distance L_2

The (15) surge tank is approximately 85% effective in isolating the section of conduit between the reservoir and surge tank from the turbine. Provided that oscillations or other undesirable effects are not introduced, the surge tank may be assumed to act as the reservoir and the relationship between the gate position and output power is given by equation II-3-72.

Due to the large capacitance placed in the circuit by the surge tank, a very low frequency oscillation mode is induced between the surge tank and the reservoir. The effect of this oscillation is investigated in Section II-3-11 to determine if one is justified in assuming that the surge tank acts as a reservoir.

Section II-3-11 The Hydro Equations for Systems with a Surge Tank

The purpose of the surge tank is to reduce the effect of

the water inertia as seen by the turbine. However, in accomplishing this, it introduces a low frequency oscillation into the system. Therefore, before the surge tank can be assumed to act as the reservoir it is necessary to determine the effect of this oscillation on the turbine output power. The effect of this oscillation is investigated by making the following assumptions: the surge tank is located beside the turbine, the throttling and pipe friction losses are directly proportional to the flow rate in the surge tank and conduit respectively.

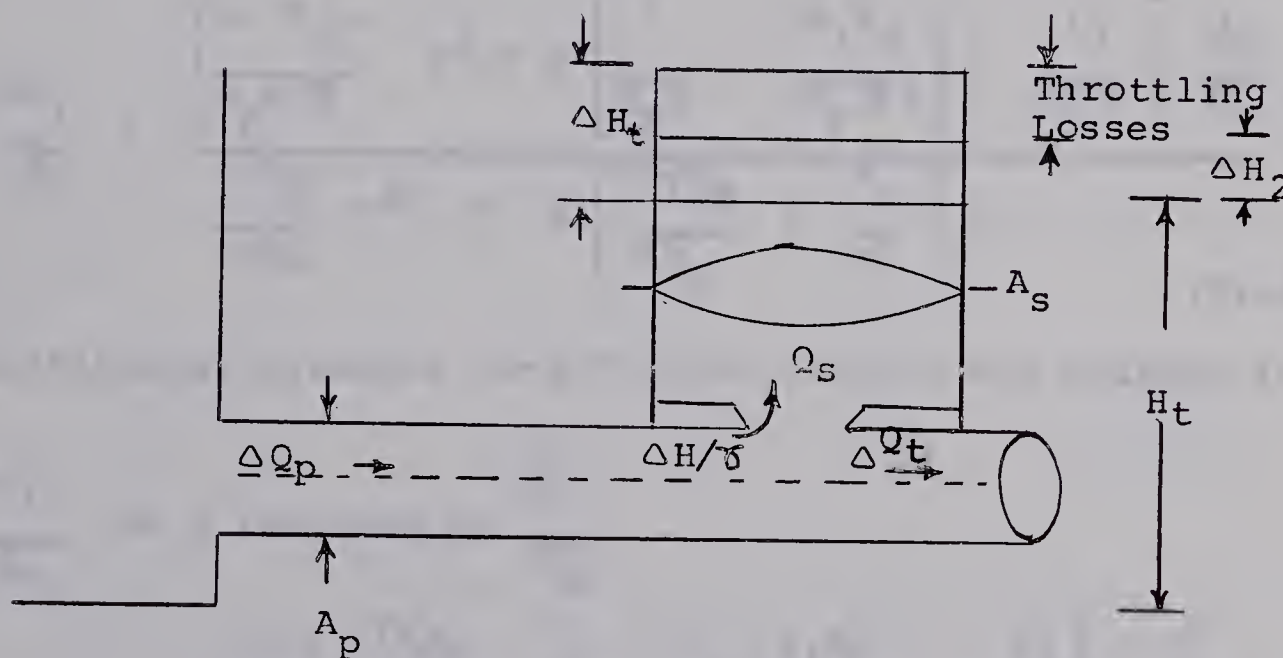


Figure II-3-29

The equations relating the pressure and flow rate variations may be determined by referring to Figure II-3-29.

$$A_s \frac{\Delta H_t}{s} = \delta Q_s + K_2 Q_s \quad (\text{II-3-73})$$

$$\rho L s \Delta Q_p + K_1 \Delta Q_p = -A_p \gamma \Delta H_t \quad (\text{II-3-74})$$

$$\frac{\Delta Q_t}{Q_o} = \sqrt{\frac{H_t}{H_o}} \frac{\Delta G}{G_o} + \frac{1}{2} \frac{G}{G_o} \sqrt{\frac{H_o}{H_t}} \frac{\Delta H_t}{H_o} \quad (\text{II-3-63})$$

$$\Delta Q_p = Q_s + \Delta Q_t \quad (\text{II-3-75})$$

Substituting equations II-3-73 and II-3-74 into II-3-75

$$\left[\rho L s + K_1 \right] \left[\frac{A_s \gamma}{\gamma + s K_2} s \Delta H_t + \Delta Q_t \right] = \gamma A_p \Delta H_t \quad (\text{II-3-76})$$

$$-\frac{\Delta H_t}{H_o} = \frac{\left[\frac{L K_2}{A_p g \gamma} s^2 + s \left[\frac{L}{A_p g} + \frac{K_1 K_2}{A_p \gamma^2} \right] + \frac{K_1}{A_p \gamma} \right] \frac{Q_o}{H_o} \frac{\Delta Q_t}{Q_o}}{\frac{L A_s}{g A_p} s^2 + s \left[\frac{K_1 A_s}{\gamma A_p} + \frac{K_2}{\gamma} \right] + 1} \quad (\text{II-3-77})$$

Substituting equation II-3-77 into II-3-63 and solving for

$$\frac{\Delta Q_t}{Q_o} \text{ as a function of } \frac{\Delta G}{G_o}$$

$$\frac{\Delta Q_t}{Q_o} = \frac{2 \frac{G_o H_t}{G H_o} \left[\frac{L A_s}{g A_p} s^2 + s \left[\frac{K_1 A_s}{\gamma A_p} + \frac{K_2}{\gamma} \right] + 1 \right]}{D} \frac{\Delta G}{G_o} \quad (\text{II-3-78})$$

$$D = \left[\begin{aligned} & \left[\frac{L Q_o K_2}{g A_p H_o \gamma} + 2 \frac{G_o}{G} \sqrt{\frac{H_t}{H_o}} \frac{L A_s}{g A_p} \right] s^2 + \frac{K_1 Q_o}{\gamma A_p H_o} + 2 \frac{G_o}{G} \sqrt{\frac{H_t}{H_o}} \\ & + s \left[\left(\frac{L}{g A_p} + \frac{K_1 K_2}{A_p \gamma^2} \right) \frac{Q_o}{H_o} + 2 \frac{G_o}{G} \sqrt{\frac{H_t}{H_o}} \left(\frac{K_2}{\gamma} + \frac{K_1 A_s}{\gamma A_p} \right) \right] \end{aligned} \right]$$

Substituting equation II-3-77 into II-3-78

$$-\frac{\Delta H_t}{H_o} = \frac{2 \frac{G_o H_t}{G H_o} \left[\frac{L K_2}{g A_p \gamma} s^2 + s \left[\frac{L}{g A_p} + \frac{K_1 K_2}{A_p \gamma^2} \right] + \frac{K_1}{A_p} \right] \frac{Q_o}{H_o}}{D} \frac{\Delta G}{G_o} \quad (\text{II-3-79})$$

Substituting equations II-3-78 and II-3-79 into equation II-3-68.

$$\begin{aligned} & \frac{L}{g A_p} \left[\sqrt{\frac{H_t}{H_o}} \frac{A_s}{\gamma} - \frac{G Q_o K_2}{G_o H_o} \right] s^2 + \sqrt{\frac{H_t}{H_o}} - \frac{G K_1 Q_o}{G_o \gamma A_p H_o} \\ \frac{\Delta P_t}{P_o} = & \frac{\left[\sqrt{\frac{H_t}{H_o}} \left(\frac{K_2}{\gamma} + \frac{K_1 A_s}{\gamma A_p} \right) - \frac{G}{G_o} \left(\frac{L}{g A_p} + \frac{K_1 K_2}{A_p \gamma^2} \right) \frac{Q_o}{H_o} \right] s}{\frac{D}{2 G_o} \left(\frac{H_o}{H_t} \right)^{3/2}} \frac{\Delta G}{G_o} \end{aligned} \quad (\text{II-3-80})$$

Equation II-3-80 may be simplified by approximating the s^2 and s^0 coefficients as follows.

The s^2 numerator coefficient

$$\sqrt{\frac{H_t}{H_o}} \frac{LA_s}{gA_p} - \frac{G Q_o L K_2}{G_o H_o g A_p \gamma} = \sqrt{\frac{H_t}{H_o}} \frac{LD_s}{gD_p} - \frac{G K_2}{G_o} T'_w$$

As $G \leq G_o$, and assuming that $H_t = H_o$, $D_s/D_p = 5 > T'_w$

Then

$$\sqrt{\frac{H_t}{H_o}} \frac{LA_s}{gA_p} - \frac{G Q_o L K_2}{G_o H_o g A_p \gamma} > \frac{L}{g} - \frac{K_2}{5\gamma}$$

Now

$$\frac{K_2}{\gamma} U_s = \text{Throttling losses in ft.}$$

The maximum throttling losses are in the neighbourhood of 40 ft. (reference 10, pp. 237).

$$\frac{K_2}{\gamma} U_s = 40 \text{ ft. and } U_s \text{ max.} = (D_p/D_s)^2 U_p \text{ max.}$$

Assuming $U_p \text{ max.} = 20' / \text{sec.}$, then $K_2/5 \text{ max.} = 10$

Therefore, if the length of the conduit is greater than 322 ft. the s^2 coefficient is always positive. Under normal operating conditions, U_s is small, therefore

$$\sqrt{\frac{H_t}{H_o}} \frac{LA_s}{gA_p} \gg \frac{G Q_o L K_2}{G_o H_o g A_p \gamma}$$

The s^1 numerator coefficient

$$\sqrt{\frac{H_t}{H_o}} \left[\frac{K_2}{\gamma} + \frac{K_1 A_s}{\gamma A_p} \right] - \frac{G L Q_o}{G_o g A_p H_o} + \frac{G K_1 K_2 Q_o}{G_o A_p \gamma^2 H_o}$$

$$\frac{K_1}{\gamma} U_p = \text{friction losses in ft.}$$

$$\frac{K_1}{\gamma} U_p = f_f \frac{L U_p^2}{D_p^2 g}$$

$$\frac{K_1}{\gamma} U_p = f_f \frac{L U_p}{D_p g} \frac{U_p}{g}$$

$$\frac{K_1}{\gamma} = f_f \frac{L U_p}{D_p g}$$

Assuming $H_t = H_o$, $(G/A_p) U_o = U_p$, $D_p = 8'$, $U_p \text{ max} = 20' / \text{sec.}$,

$$f_f = 0.014, A_p = G_o$$

$$\sqrt{\frac{H_t}{H_o}} \left[\frac{K_2}{\gamma} + \frac{K_1 A_s}{\gamma A_p} \right] - \frac{G}{G_o} \left[\frac{L Q_o}{g A_p H_o} + \frac{K_1 K_2 Q_o}{A_p \gamma^2 H_o} \right]$$

$$= \frac{K_2}{\gamma} \left[1 - \frac{G K_1 Q_o}{G_o A_p \gamma H_o} \right] + \frac{K_1 A_s}{\gamma A_p} - \frac{G L Q_o}{G_o g A_p H_o}$$

$$= \frac{K_2}{\gamma} \left[1 - f_f \frac{U_p^2 L}{g D_p H_o} \right] + \frac{U_p L}{g} \left[\frac{f_f A_s}{D_p A_p} - \frac{1}{H_o} \right]$$

$$= \frac{K_2}{\gamma} \left[1 - 1.4 \times 10^{-4} \right] \frac{L}{H_o} + \frac{U_p L}{g} \left[4.375 \times 10^{-2} - \frac{1}{H_o} \right]$$

Therefore, if the head H_o is greater than 22 ft. and L/H_o 7.142, the s^1 term will be positive.

The s^0 numerator coefficient

Assuming $H_t = H_o$, $(G/A_p)Q_o = Q_p$, $f_f = 0.014$, $D_p = 8'$, $G_o = A_p$

$$\sqrt{\frac{H_t}{H_o}} - \frac{G K_1 Q_o}{G_o A_p \gamma H_o} = 1 - \frac{f_f L U_p U_p}{\gamma D_p g A_p H_o}$$

$$= 1 - 1.4 \times 10^{-4} \frac{L}{H_o}$$

If $L/H_o < 100$, then

$$\sqrt{\frac{H_t}{H_o}} - \frac{G K_1 Q_o}{G_o A_p \gamma H_o} = \sqrt{\frac{H_t}{H_o}}$$

By approximating the s^2 and s^0 terms of the numerator and denominator, equation II-3-80 is reduced to equation II-3-81.

It has been shown that the terms $(G/G_o) (Q_o/H_o) (L/gA_p) (K_2/\gamma)$ and $(G/G_o) (K_1/A_p \gamma) (Q_o/H_o)$ may be neglected with respect to the terms $(H_t/H_o) (LA_s/gA_p)$ and $\sqrt{H_t/H_o}$. It can also be shown that the former terms may be neglected in the denominator.

$$\frac{\sqrt{\frac{H_t}{H_o}} \left[\frac{K_2}{\gamma} + \frac{K_1 A_s}{\gamma A_p} \right] - \frac{G}{G_o} \left[\frac{L}{g A_p} + \frac{K_1 K_2}{A_p \gamma^2} \right] \frac{Q_o}{H_o}}{\sqrt{\frac{H_t}{H_o}} - \frac{LA_s}{g A_p}} s \quad \text{(II-3-81)}$$

$$\frac{\Delta P_t}{P_o} = \left(\frac{H_t}{H_o} \right)^{3/2} \frac{+ s^2 + \frac{g A_p}{L A_s}}{+ s^2 + \frac{g A_p}{L A_s}} \frac{\Delta G}{G_o}$$

+

$$\begin{aligned}
 & \left[\frac{L}{gA_p} + \frac{K_1 K_2}{A_p \gamma^2} \right] \frac{Q_c}{H_c} + 2 \frac{G_c}{G} \left[\sqrt{\frac{H_t}{H_c}} \frac{K_2}{\gamma} + \frac{K_1 A_s}{\gamma A_p} \right] s \\
 & 2 \frac{G_o}{G} \sqrt{\frac{H_t}{H_o}} \frac{LA_s}{gA_p}
 \end{aligned}$$

The coefficients of the s^1 terms are very small and therefore, two poles and two zeros are introduced at $\sqrt{gA_p/LA_s}$. The poles and zeros cancel each other, thus the effect of the low frequency oscillation between the surge tank and reservoir may be omitted.

The surge tank may be assumed to act as the reservoir, with equation II-3-72, giving the relationship between the gate position and the output power.

Section II-3-12 The Effect of the Higher Harmonics in the Pressure Variation

The linearization of equation II-3-2 implies that the harmonic content in the pressure variation should decrease as smaller gate movements are considered. This was checked by graphical means and was not found to be the case. In this section, the harmonic components will be assumed to exist and their effect on system stability will be considered.

The system is considered linear with a third harmonic signal being applied to represent the harmonic components generated by the non-linearity (refer to Figure II-3-30).

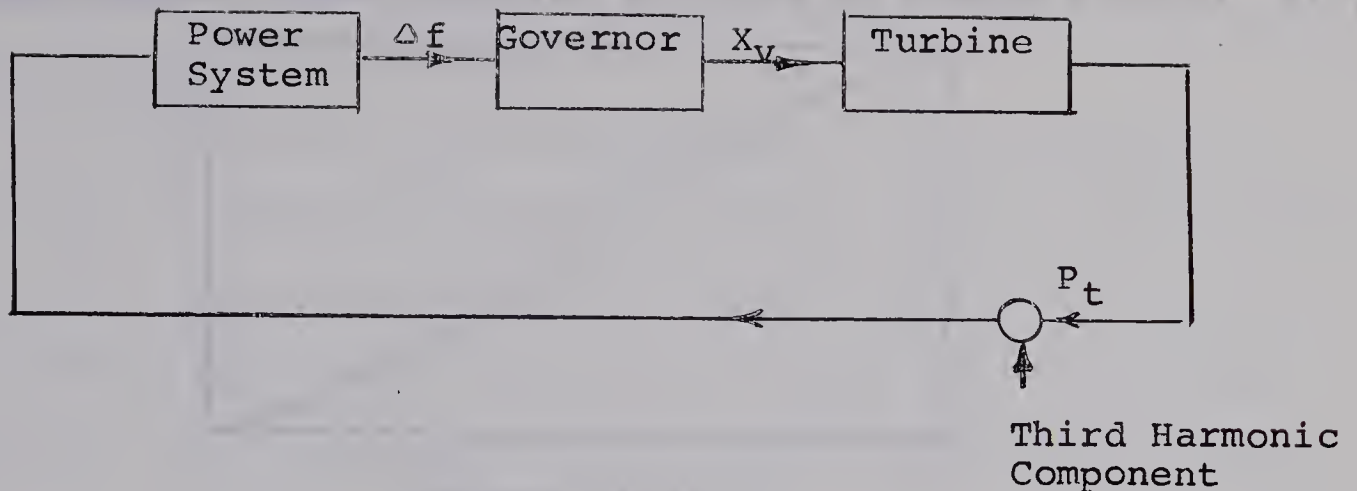


Figure II-3-30

The pressure variations exhibit half wave symmetry (Figure II-3-22), indicating that its constituents are mainly a fundamental and odd harmonics. Only the third harmonic is considered as it is the largest component.

Assuming the power system does not greatly attenuate the third harmonic component relative to the fundamental, the third harmonic will appear in the frequency deviation (Δf). It will not appear, however, in the gate movement as the governor is not capable of responding to a frequency oscillation of 0.3 cycles per second (reference 10, page 94). The oscillation will not have a large effect on the power system as its natural frequency is approximately 1.0 cycles per second. (18)

In view of the above and the fact that the linearization appears sound mathematically, the harmonic components in the pressure variation are ignored.

Section II-3-13 The Effect of the Turbine Wicket Gate

Non-linearity

The turbine gate position is a non-linear function of

the servo-motor valve position as shown by Figure II-3-31. (10)

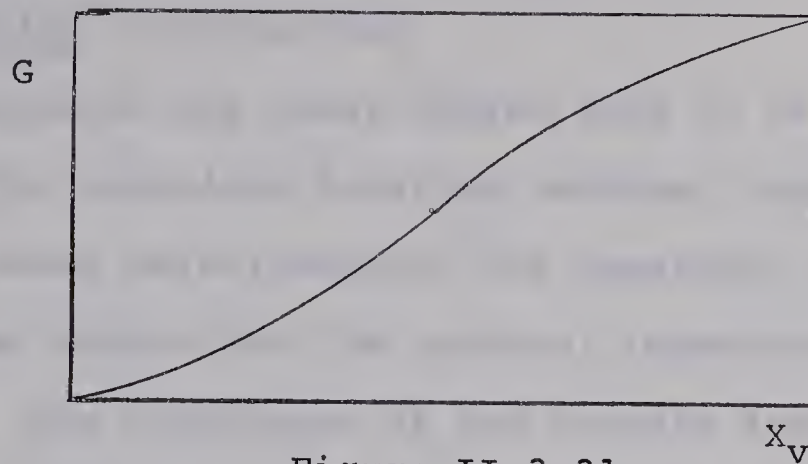


Figure II-3-31

A step load change of 50% rated output power was applied to the governing loop with and without the non-linearity. The resulting oscillations are shown in Figures II-3-32 and II-3-33.

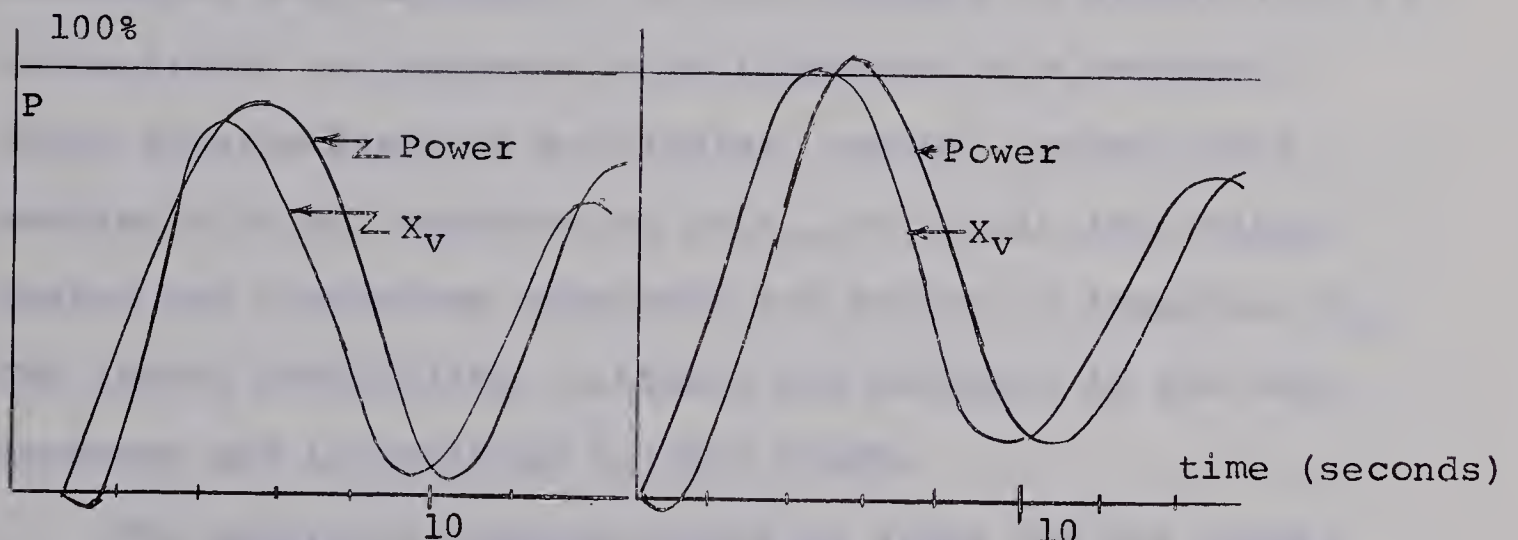


Figure II-3-32

Figure II-3-33

As both oscillations are damped at almost an equal rate, the non-linearity is seen to have little effect on the stability of the loop and is neglected.

Section II-3-14 Conclusion

It is found that the compressibility of the water and the elasticity of the conduit do not effect the turbine power output, and that the surge tank isolates the turbine from the reservoir.

Section II-4 The Load Model

Section II-4-0 Introduction

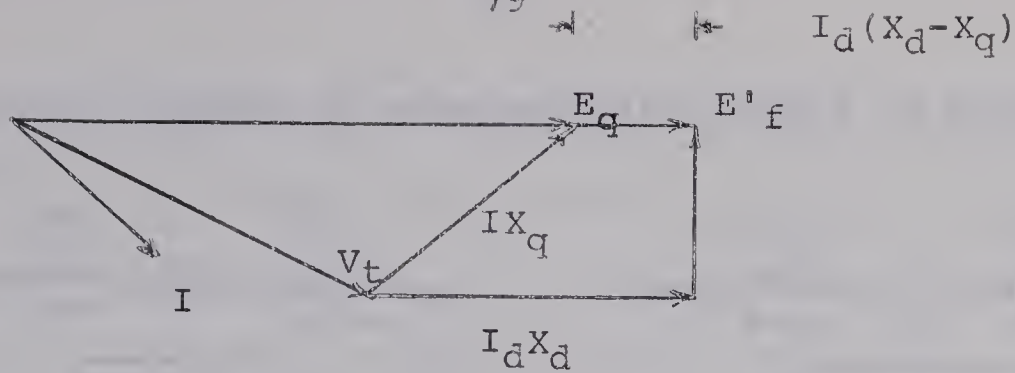
To represent the power system load it is necessary to determine the equations relating voltage, impedance and current. To obtain these relationships, the generator is considered as a voltage source with an internal impedance looking into a network. The impedances of the network are assumed constant as the load damping action is accounted for in equation II-2-9.

The internal impedance of the salient pole generator is a non-linear function of the machine's internal and synchronizing torque angle as the direct and quadrature reactances are different. In the interest of simplicity, it is desirable to represent this impedance as a constant. Three alternatives are available: assume a wound rotor machine with an impedance X_d or X_q , or obtain the voltage behind the quadrature reactance and assume an impedance X_q . The latter possibility, although not perfect, is the most accurate and is utilized in this study.

The generator representation is given and the errors are discussed in Section II-4-1. The equations are then developed in Section II-4-2. The current equations, along with equations II-1-66, II-1-70 and II-1-73 allow the generator output or load power to be determined.

Section II-4-1 The Generator Representation

To represent generator saliency, one must consider the voltage behind the quadrature reactance (refer to Figure II-4-1). (6)



$$\begin{aligned} \text{Where } E_q &= I_f X_{ad} + I_{kd} X_{ad} - I_d (X_d - X_q) \\ &= E'_f - I_d (X_d - X_q) \end{aligned} \quad (\text{II-1-53})$$

Figure II-4-1

The above representation yields a correct equivalent circuit if the generator and its load are considered to be isolated from the system. Therefore the non-linearity of the internal reactance as a function of the internal torque angle is eliminated. However, if two machines are connected together through a mutual load, the non-linearity of the impedance of the second machine as seen by the first is in error. Therefore the non-linearity of the internal impedance as a function of the synchronizing torque angle still exists. This may be seen as follows (refer to Figures II-4-2, II-4-3, II-4-4.)

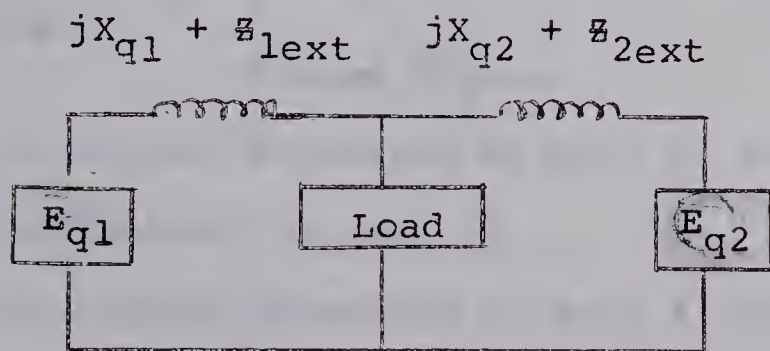


Figure II-4-2

Applying the theorem of superposition (refer to Figure II-4-3)

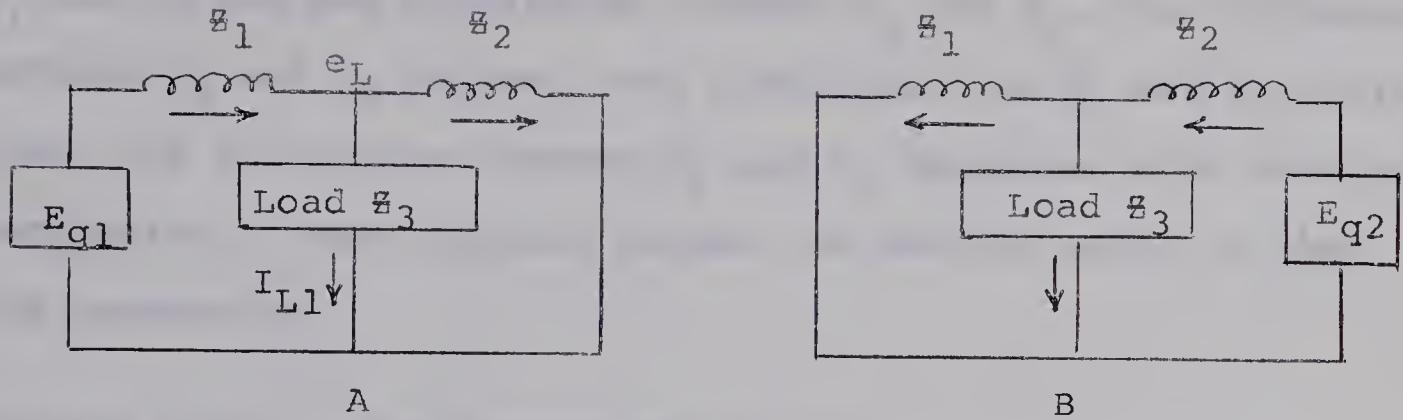


Figure II-4-3

The vector diagram for Figure II-4-3-A is shown by Figure II-4-4, assuming $\bar{Z}_1 = jX_1$, $\bar{Z}_2 = jX_2$ and $\bar{Z}_3 = R$.

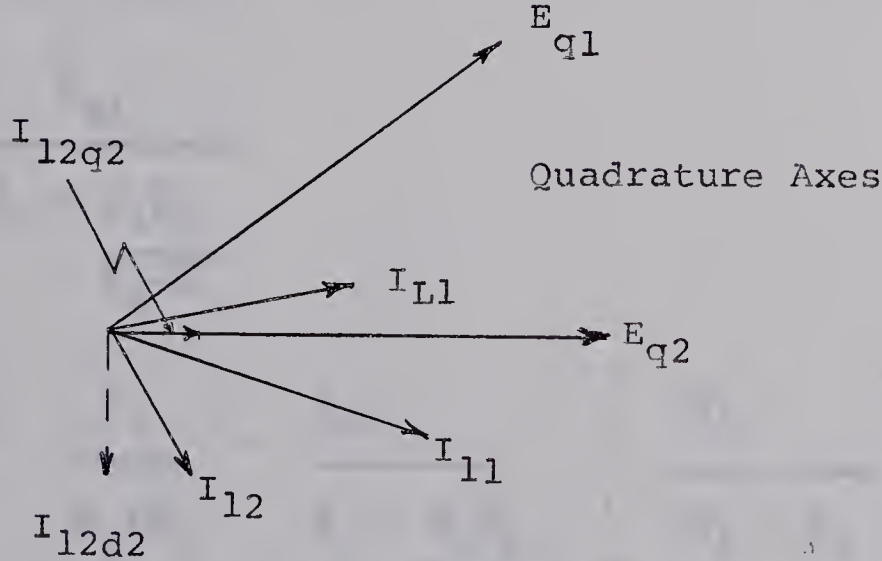


Figure II-4-4

I_{12q2} is the current generated by unit #1 along the quadrature axis of unit #2

I_{12d2} is the current generated by unit #1 along the direct axis of unit #2.

As the reactance associated with I_{12d2} should be X_d and not X_q , this representation decreases the impedance and increases the synchronizing power between units.

The magnitude of the error depends on the values of $\bar{z}_1$ and $\bar{z}_2$ and the difference between X_d and X_q . The difference between X_d and X_q becomes less significant as $\bar{z}_1$ and $\bar{z}_2$ increase. Also, the difference between X_d and X_q decreases with machine saturation. These factors reduce the model's error to very low proportions.

Section II-4-2 The Current Equations

Referring to Figure II-4-3, equations II-4-1 to II-4-4 are written, in terms of vectors, by inspection.

$$\bar{I}_{11} = \frac{\bar{E}_{q1}}{\frac{\bar{z}_1 + \bar{z}_2\bar{z}_3}{\bar{z}_2 + \bar{z}_3}} \quad (\text{II-4-1})$$

$$\bar{I}_{22} = \frac{\bar{E}_{q2}}{\frac{\bar{z}_2 + \bar{z}_1\bar{z}_3}{\bar{z}_1 + \bar{z}_3}} \quad (\text{II-4-2})$$

$$\bar{I}_{21} = \bar{I}_{22} \frac{\bar{z}_3}{\bar{z}_1 + \bar{z}_3} = \frac{\bar{E}_{q2}}{\frac{\bar{z}_2 + \bar{z}_1\bar{z}_3}{\bar{z}_1 + \bar{z}_3}} \left[\frac{\bar{z}_3}{\bar{z}_1 + \bar{z}_3} \right]$$

$$\bar{I}_{21} = \frac{\bar{E}_{q2}}{\frac{\bar{z}_1 + \bar{z}_2 + \bar{z}_1\bar{z}_2}{\bar{z}_3}} \quad (\text{II-4-3})$$

$$\bar{I}_{12} = \frac{\bar{E}_{q1}}{\bar{Z}_1 + \bar{Z}_2 + \frac{\bar{Z}_1 \bar{Z}_2}{\bar{Z}_3}} \quad (\text{II-4-4})$$

Let

$$\bar{Z}_{11} = \frac{\bar{Z}_1 + \bar{Z}_2 \bar{Z}_3}{\bar{Z}_2 + \bar{Z}_3} = Z_{11} \angle \theta_{11} \quad (\text{II-4-5})$$

$$\bar{Z}_{12} = \bar{Z}_{21} = \frac{\bar{Z}_1 + \bar{Z}_2 + \bar{Z}_1 \bar{Z}_3}{\bar{Z}_3} = Z_{12} \angle \theta_{12} \quad (\text{II-4-6})$$

$$\bar{Z}_{22} = \frac{\bar{Z}_2 + \bar{Z}_1 \bar{Z}_3}{\bar{Z}_1 + \bar{Z}_3} = Z_{22} \angle \theta_{22} \quad (\text{II-4-7})$$

$$\bar{I}_1 = \frac{\bar{E}_{q1}}{\bar{Z}_{11}} + \frac{\bar{E}_{q2}}{\bar{Z}_{12}} \quad (\text{II-4-8})$$

$$\bar{I}_2 = \frac{\bar{E}_{q2}}{\bar{Z}_{22}} + \frac{\bar{E}_{q1}}{\bar{Z}_{12}} \quad (\text{II-4-9})$$

Through the use of a resolver, it is possible to represent E_{q1} and E_{q2} as vectors. This makes it possible to obtain the direct and quadrature currents correctly and also allows a vector notation to be attached to them. However, to obtain the power output it is necessary to multiply these currents by the direct and quadrature voltages, which are represented as scalars (refer to equation II-1-73). To

allow this multiplication to be carried out correctly, zero angle must be attached to the currents. This is accomplished by considering each generator to be at zero reference and the other generator to be leading or lagging it. This has no detrimental effects on the simulation as it yields correct generator output power, which in turn means correct frequency and synchronizing torque angle. On this basis equations II-4-8 and II-4-9 are rewritten as equations II-4-10 and II-4-11, assuming generator #1 to be leading generator #2 by an angle δ .

$$\bar{I}_1 = \frac{\bar{E}_{q1}}{\bar{Z}_{11}} - \frac{\bar{E}_{q2}}{\bar{Z}_{12}} \angle -\delta \quad (\text{II-4-10})$$

$$\bar{I}_2 = \frac{\bar{E}_{q2}}{\bar{Z}_{22}} - \frac{\bar{E}_{q1}}{\bar{Z}_{12}} \angle \delta \quad (\text{II-4-11})$$

$$\bar{I}_1 = \frac{E_{q1}}{Z_{11}} \left[\cos(-\theta_{11}) + j \sin(-\theta_{11}) \right] \\ - \frac{E_{q2}}{Z_{12}} \left[\cos(-\delta - \theta_{12}) + j \sin(-\delta - \theta_{12}) \right]$$

$$\bar{I}_1 = \frac{E_{q1}}{Z_{11}} \left[\cos \theta_{11} - j \sin \theta_{11} \right] \quad (\text{II-4-12})$$

$$- \frac{E_{q2}}{Z_{12}} \left[\left[\cos \delta \cos \theta_{12} - \sin \delta \sin \theta_{12} \right] + j \left[\sin \delta \cos \theta_{12} + \cos \delta \sin \theta_{12} \right] \right]$$

The direct and quadrature axis components

$$\bar{I}_{d1} = -j \frac{E_{q1}}{\bar{Z}_{11}} \sin \theta_{11} - j \frac{E_{q2}}{\bar{Z}_{12}} [\sin \delta \cos \theta_{12} + \cos \delta \sin \theta_{12}] \quad (\text{II-4-13})$$

$$\bar{I}_{q1} = \frac{E_{q1}}{\bar{Z}_{11}} \cos \theta_{11} - \frac{E_{q2}}{\bar{Z}_{12}} [\cos \delta \cos \theta_{12} - \sin \delta \sin \theta_{12}] \quad (\text{II-4-14})$$

$$\begin{aligned} \bar{I}_2 = & \frac{E_{q2}}{\bar{Z}_{22}} [\cos(-\theta_{22}) - j \sin \theta_{22}] \\ & - \frac{E_{q1}}{\bar{Z}_{12}} [\cos \delta \cos \theta_{12} + \sin \delta \sin \theta_{12}] \\ & - j [\sin \delta \cos \theta_{12} - \cos \delta \sin \theta_{12}] \end{aligned} \quad (\text{II-4-15})$$

The direct and quadrature axis components

$$\bar{I}_{q2} = \frac{E_{q2}}{\bar{Z}_{22}} \cos \theta_{22} - \frac{E_{q1}}{\bar{Z}_{12}} [\cos \delta \cos \theta_{12} + \sin \delta \sin \theta_{12}] \quad (\text{II-4-17})$$

$$\bar{I}_{d2} = -j \frac{E_{q2}}{\bar{Z}_{22}} \sin \theta_{22} - j \frac{E_{q1}}{\bar{Z}_{12}} [\sin \delta \cos \theta_{12} - \cos \delta \sin \theta_{12}] \quad (\text{II-4-16})$$

As the currents do not change instantaneously when a load disturbance is applied to the system, equations II-4-13, II-4-14, II-4-16 and II-4-17 only represent the currents under steady state operating conditions. To represent this transient condition, the direct axis time constant is assumed to be equal to the short circuit time constant and the quadrature axis time constant is assumed to be zero. This is only considered to be an approximation as the time constants associated with the direct and quadrature axis vary with the external load.

It is necessary to divide the equations for the direct axis currents by $-j$ to remove the vector notation and place them on the same basis as the direct axis voltages.

Rewriting equations II-4-13, II-4-14, II-4-16 and II-4-17 in per unit and including the above mentioned modifications, one obtains the following equations:

$$I_{d1} = \frac{1}{T_d' s + 1} \left[\frac{E_{q1} \sin \theta_{11}}{Z_{11}} \right] - \frac{E_{q2}}{Z_{12}} \left[\sin \delta \cos \theta_{12} + \cos \delta \sin \theta_{12} \right] \quad (\text{II-4-18})$$

$$I_{q1} = \frac{E_{q1}}{Z_{11}} \cos \theta_{11} - \frac{E_{q2}}{Z_{12}} \left[\cos \delta \cos \theta_{12} - \sin \delta \sin \theta_{12} \right] \quad (\text{II-4-14})$$

$$I_{d2} = \frac{1}{T_d' s + 1} \left[\frac{E_{q2} \sin \theta_{22}}{Z_{22}} \right] + \frac{E_{q1}}{Z_{12}} \left[\sin \delta \cos \theta_{12} - \cos \delta \sin \theta_{12} \right] \quad (\text{II-4-19})$$

$$I_{q2} = \frac{E_{q2}}{Z_{22}} \cos \theta_{22} - \frac{E_{q1}}{Z_{12}} \left[\cos \delta \cos \theta_{12} + \sin \delta \sin \theta_{12} \right] \quad (\text{II-4-16})$$

The synchronizing torque angle is obtained from the frequency deviation which is expressed in per unit. As there are $360f_0$ degrees per second squared in one per unit cycle per second, the individual machine's datum angle is given by equation II-4-20.

$$\phi = \frac{360 f}{s} \quad (\text{II-4-20})$$

The torque angle between machines

$$\delta = \phi_1 - \phi_2 = \frac{360 (f_1 - f_2)}{s} \quad (\text{II-4-21})$$

The simulated equations are II-4-14, II-1-16, II-1-18 and II-1-19 expressed in per unit, and II-4-21 expressed in degrees.

Section II-4-3 Conclusion

The generator may be accurately represented by considering it as a voltage source (E_q) with an internal impedance X_q .

Section II-5 The Controller Model

Section II-5-0 Introduction

To simulate the controller, it is necessary to determine the relationships between the variables of its operation. This is most easily accomplished by understanding how the unit operates, allowing approximations to be made and then developing the relationships. Therefore a description of the controller is given in Section II-5-1, approximations are made in Section II-5-2 and a method of simulation is decided upon in Section II-5-3.

Section II-5-1 The Description of the Controller's Operation

The functional purpose of the controller is to provide correct time and load sharing between units with correct frequency being a by-product. The operation of the controller is shown in Figure II-5-1.

As the load sharing feature is introduced by the use of a bias time signal and does not directly effect the controller's operation, it is ignored at this point to facilitate the operational description of the unit.

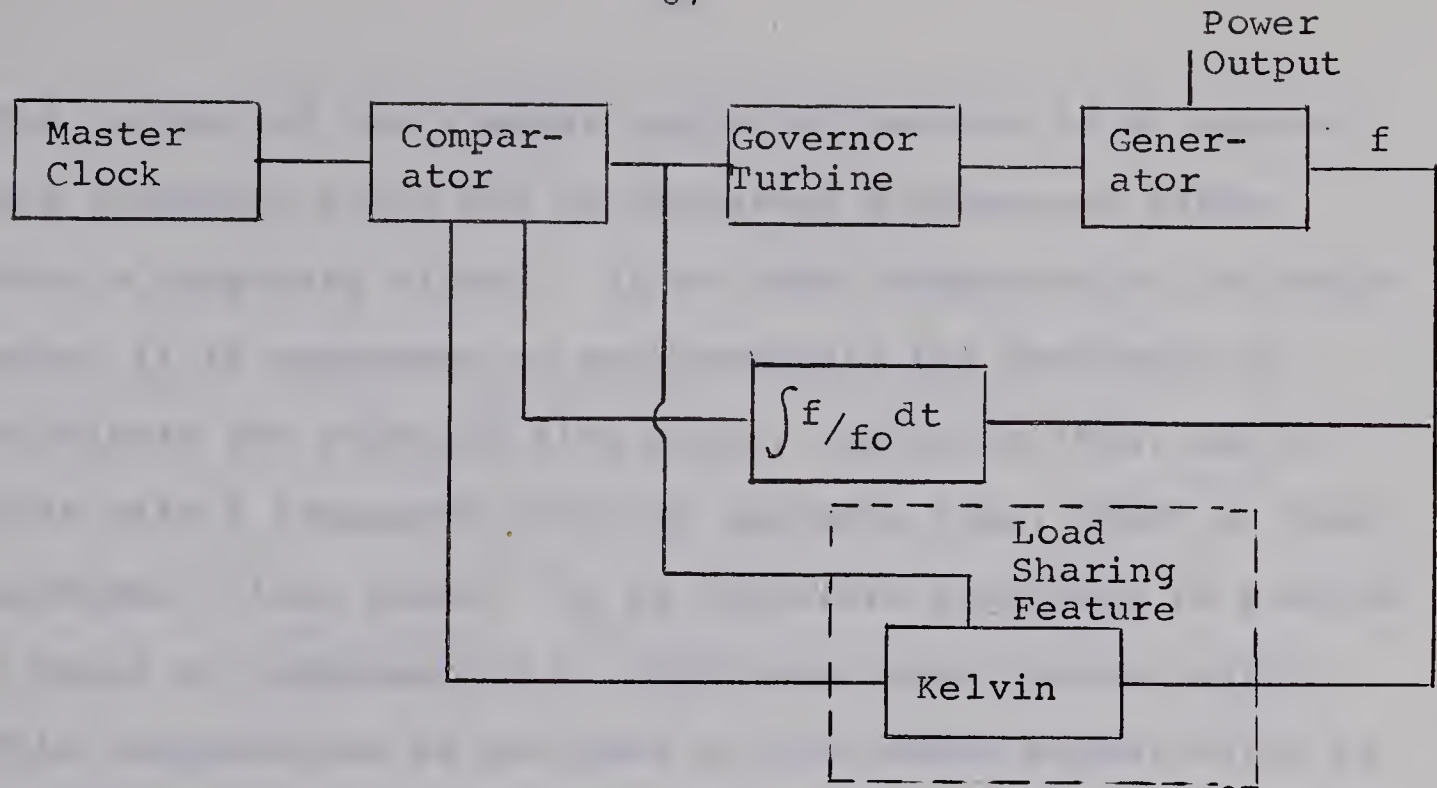


Figure II-5-1

The master clock, located at the central control station, sends out a time signal of one second duration every ten seconds. The front and trailing edges of this signal are compared to the electrical time, thereby determining whether the latter is fast or slow respectively. If, at the n^{th} sampling instant, the electrical time is in error, two signals are applied to the governor. The first adjusts the permanent speed droop curve by changing the speeder spring setting and the second is applied to the turbine wicket gates through the compensating dashpot. When the time is correct at the $(n + 1)^{\text{th}}$ sampling instant, the dashpot signal is removed, however, the speeder spring setting is retained. The applied signals are of constant magnitude and not proportional to the time error.

The purpose of these signals is most easily seen by considering the effect of the first without the second. A time error necessitates the past or present existence of a frequency error, as time is the integral of frequency.

The purpose of the speeder spring adjustment is to correct the frequency error and is therefore a permanent rather than a temporary signal. If no other compensation is available, it is necessary to over-regulate the frequency to eliminate the existing time error. In doing this, one is left with a frequency error of opposite sign, which in turn produces a time error. It is therefore necessary to provide a means of compensation to counteract this hunting effect. This compensation is provided by the second signal which is applied over a thirteen second interval. The first signal adjusts the speed droop curve to correct the frequency error, while the second causes the unit to overspeed to correct the time error.

The comparison between the master clock and electrical time is made through the use of a synchronous motor, a system of relays and mechanical linkages. The synchronous motor rotates an arm in the control unit once every ten seconds. Assuming correct time, the arm will pass over the top position when the signal from the master clock is received. If the time is slow the arm will not have reached the top position and vice versa, when the clock signal is received. Any error is sensed through the system of relays and governor action ensues. It is to be noted that the relay contacts are mounted on a quadrant which is not rigidly attached to the motor frame (refer to Figure II-5-2). This is of prime importance to the load sharing feature and is utilized to provide a degree of stabilization.

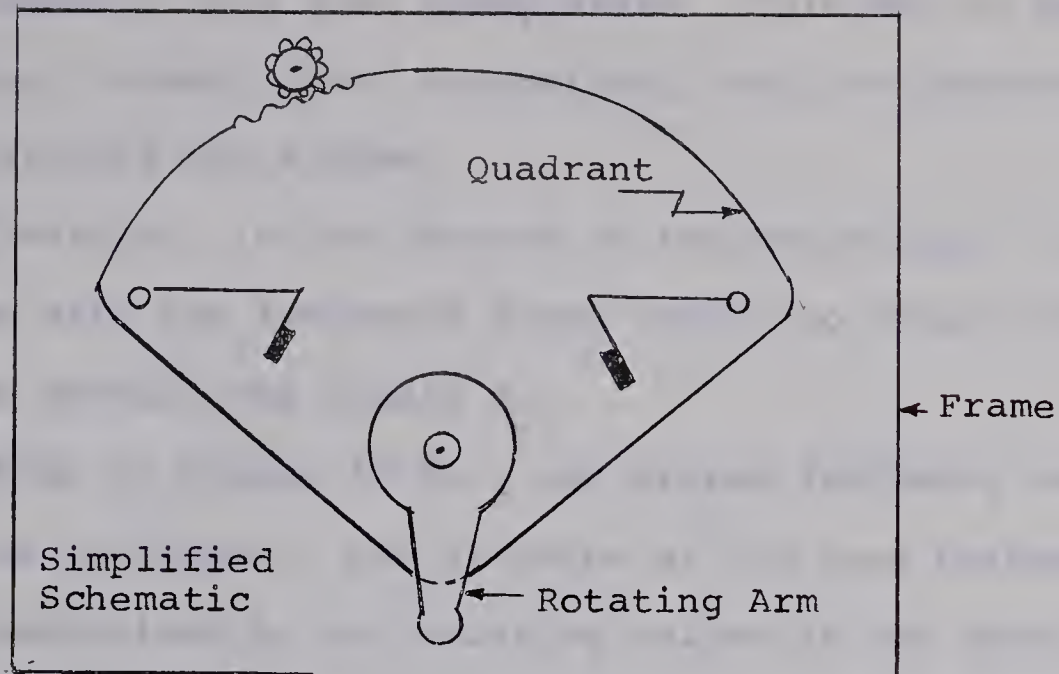


Figure II-5-2

Load sharing between units

The generator output is measured by a Kelvin balance whose force is counterbalanced against a spring which is indirectly connected to the quadrant. If the forces are not equal, a motor is energized and the quadrant is rotated, thereby making the quadrants' position directly proportional to the load being carried by the unit. The position of the quadrant affects the time error as seen by the master clock, as with an increase in load it advances the electrical time. If one machine is carrying more than its fair share of load, its time appears fast with respect to the other units. The controller, in correcting this bias time error, forces the unit to drop load as its speed droop curve is lowered. If it is desired for one unit to carry other than its share of load, it is put on percentage load control by adjusting the force of the Kelvin balance.

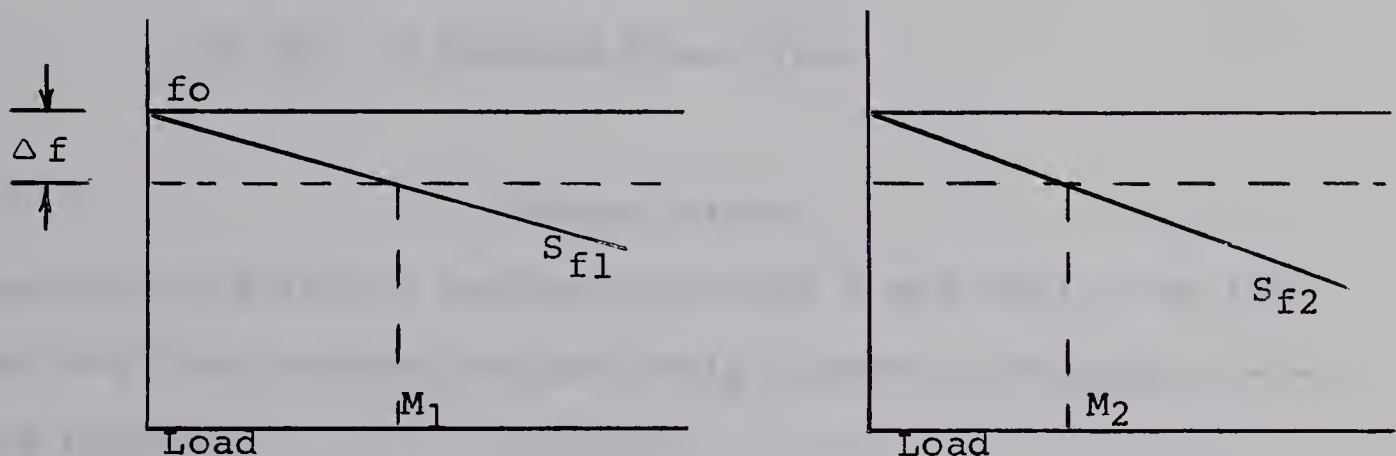
The controller converts the load sharing between units,

from a frequency to a time droop basis. This may be seen by assuming: steady state conditions, that two generating units constitute the system.

Load sharing, in the absence of the controller, is in accordance with the frequency droop (refer to Figure II-5-3). Assume the system load equals M .

As shown by Figure II-5-3, the system frequency decreases as the load increases. The division of the load between units is determined by the relative values of the speed droops (S_{f1} , S_{f2}).

$$M_1 = \frac{S_{f2}}{S_{f1} + S_{f2}} M \quad M_2 = \frac{S_{f1}}{S_{f1} + S_{f2}} M \quad (\text{II-5-1})$$



$$M = M_1 + M_2$$

Figure II-5-3

Load sharing with a controller and 100% load control (refer to Figures II-5-4 and II-5-5).

The controller shifts the speed droop curves in the vertical direction to yield correct frequency in its attempt to provide correct time. As $S_{t1} = S_{t2}$, $M_1 = M_2$, and the curves are raised proportionally to the droop settings.

$$\Delta f_1 = S_{f1} M_1 \quad \Delta f_2 = S_{f2} M_2 \quad (\text{II-5-2})$$

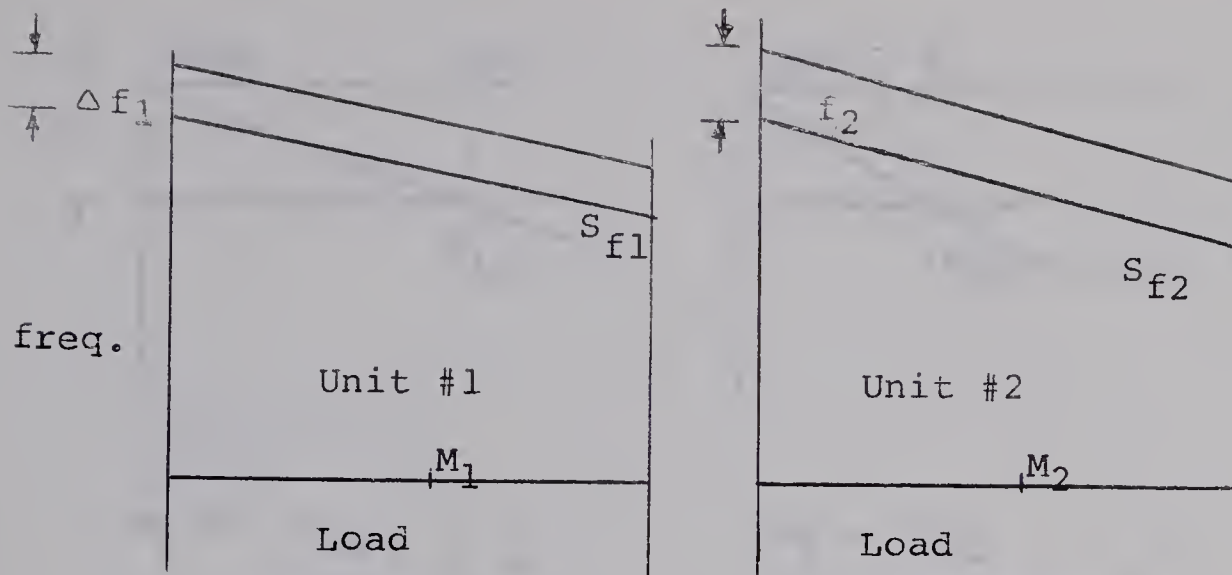
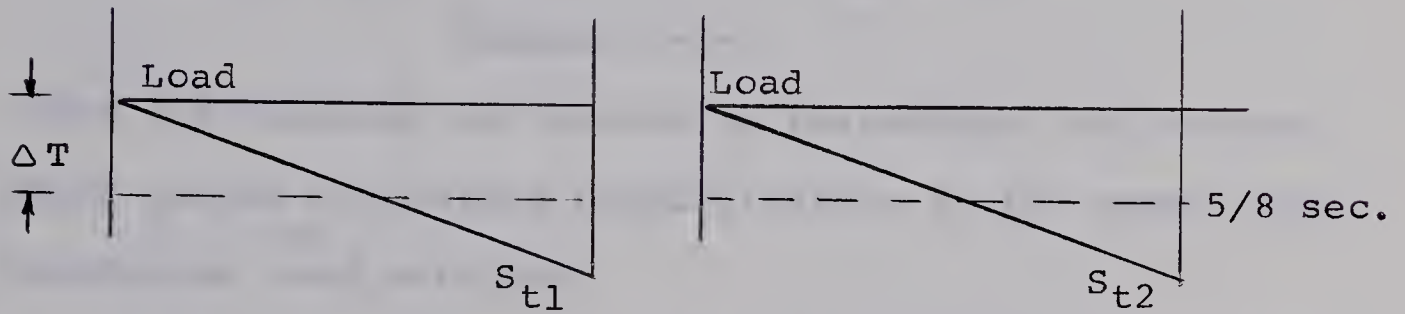


Figure II-5-4



$5/8 \text{ sec.} = \text{Maximum Time Droop}$

Figure II-5-5

Load sharing with a controller, Unit 1 and Unit 2 on 100% and 60% load control respectively (refer to Figures II-5-6 and II-5-7).

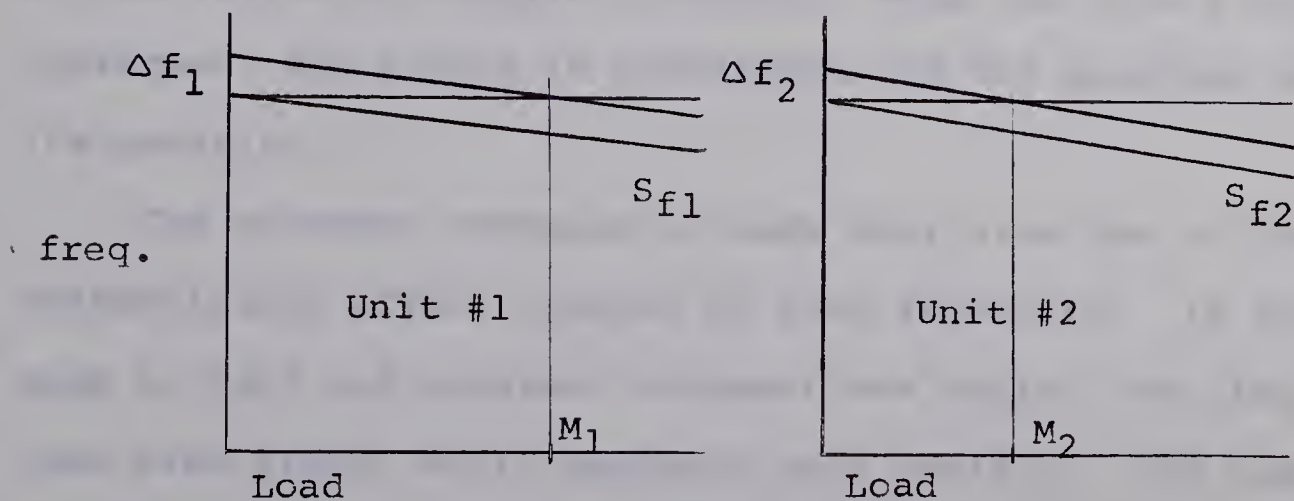
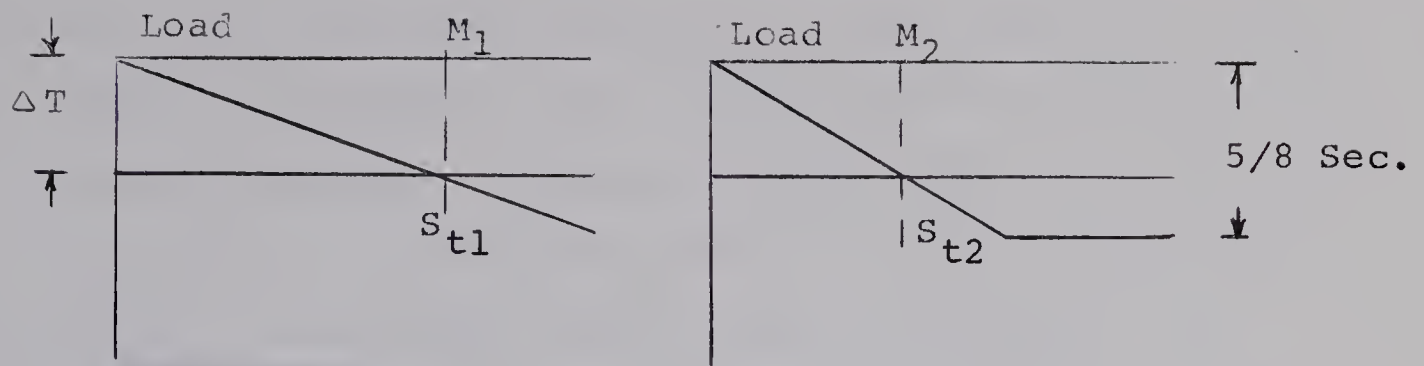


Figure II-5-6



$$M_1 = \frac{S_{t2}}{S_{t1} + S_{t2}} M \qquad M_2 = \frac{S_{t1}}{S_{t1} + S_{t2}} M$$

Figure II-5-7

When the machines are placed on percentage load control, the droop curves are raised proportionally to the speed droop and percentage load settings.

$$\Delta f_1 = \frac{S_{f1} S_{t2}}{S_{t1} + S_{t2}} M \qquad \Delta f_2 = \frac{S_{f2} S_{t1}}{S_{t1} + S_{t2}} M \qquad (\text{II-5-3})$$

Stabilization is introduced through the quadrant movement.

If a time error exists at the n^{th} sampling instant, the Kelvin is unbalanced and the quadrant is rotated very slightly to reduce the chances of a time error existing at the following sampling instant. When the time error is corrected, the Kelvin is rebalanced and the quadrant resumes its position.

The quadrant movement is made very slow due to the destabilizing effect created by load rejection. If the load were to fall and quadrant movement was rapid, the time droop load bias signal would decrease very rapidly. The time would appear slow and the controller would call for a raise signal thus assisting overspeed and possible loss of stability. The

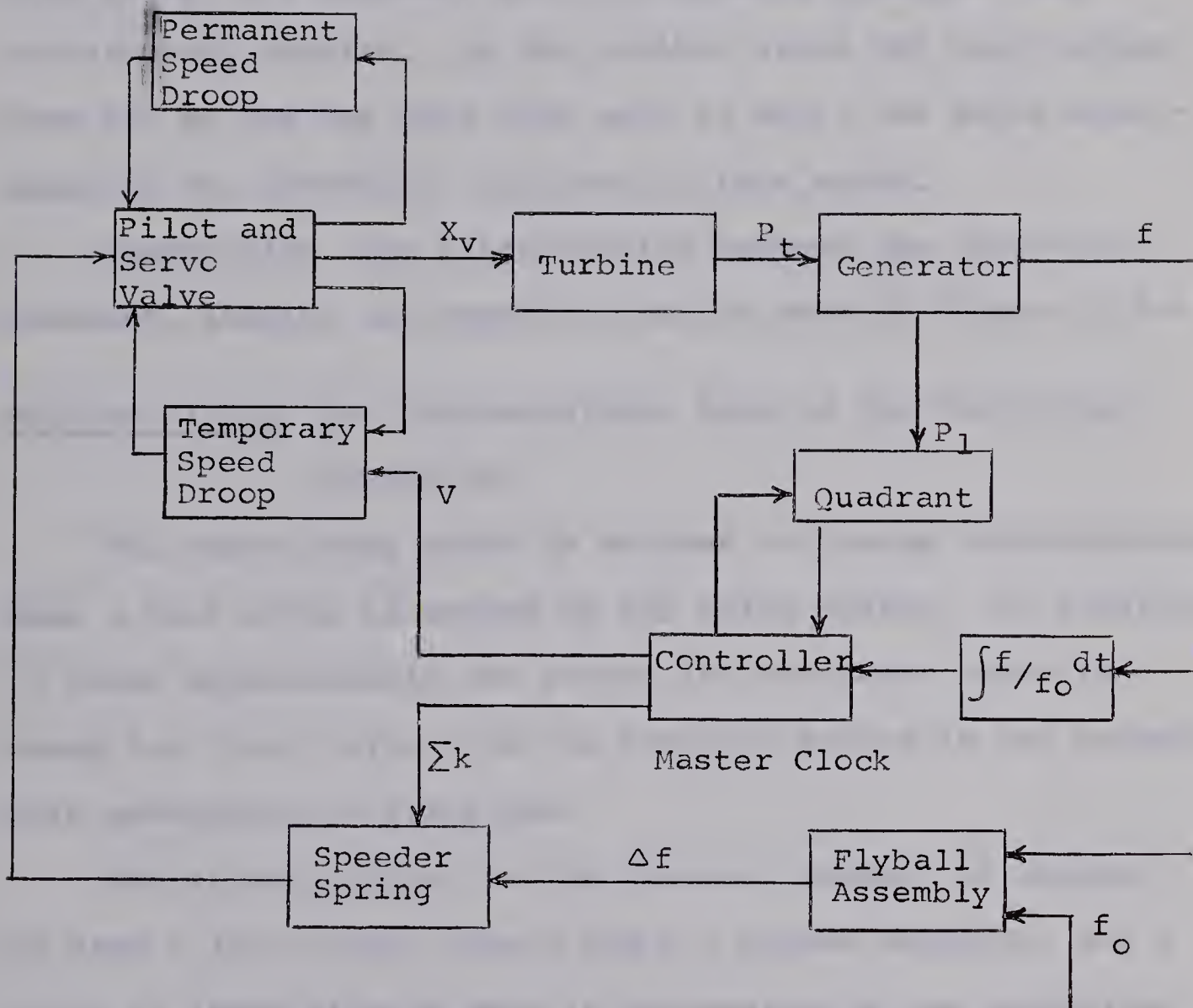


Figure II-5-8

optimum rate of quadrant movement would therefore depend on the day to day load characteristics of the individual units. Mr. Gaskell, of Calgary Power Limited, tried different rates of quadrant movement. He found that the rate of movement was not critical and allowing the quadrant to move through its time droop ($5/8$ seconds) in $2-3/4$ minutes produced very satisfactory results. As the optimum value not only varies from day to day but also from unit to unit, the value determined by Mr. Gaskell is utilized in this study.

Summarizing, the interrelation between the controller, governor, turbine and generator may be seen by Figure II-5-8.

Section II-5-2 The Approximations Made to the Controller Operation

The speed droop curve is assumed to change instantaneously when a time error is sensed by the relay system. In practice, it takes approximately one second for the droop curve to reach its final value. As the sampling period is ten seconds, this assumption is justified.

The signal applied to the governor dashpot is assumed to have a $10m$, rather than a $10m + 3$ second duration, for m raise or lower signals made in succession by the controller. The error incorporated by the above assumptions becomes small when $m \geq 3$.

The stabilization introduced through the quadrant movement is assumed to partially counteract the instability effect resulting from backlash in the mechanical linkages. A small amount of positive feedback ($1/50$ second) is utilized to

increase the time signal as seen by the controller, to simulate the backlash.

The controller is assumed to have a dead zone of 1/20 of a second.

Section II-5-3 The Simulation of the Controller

The operation of the controller is non-linear as was shown previously in Section II-5-1 and II-5-2. As an accurate linear approximation was not evident, the non-linearities were simulated directly. The method utilized to simulate these non-linearities is now given. The representation of the time error signal as seen by the controller is presented first, followed by the representation of the controller action on the time error signal. The controller action on the governor is then converted to equivalent frequency error signals.

The electrical time as seen by the controller

$$T' = \int \frac{f}{f_o} dt + \int (Load) + \int (Backlash) \quad (II-5-4)$$

The time error as seen by the controller

$$\Delta T' = \int \left[\frac{f_o + \Delta f}{f_o} \right] dt + \int (Load) + \int (Backlash) - t$$

$$\Delta T' = \int \frac{\Delta f}{f_o} dt + \int (Load) + \int (Backlash) \quad (II-5-5)$$

The Load Bias Time Signal

Assuming a step load change from no-load to full load, the quadrant will move to its extreme position in 2 3/4

minutes. The quadrant is driven by a motor and its position is directly proportional to the time droop (Figure II-5-9).

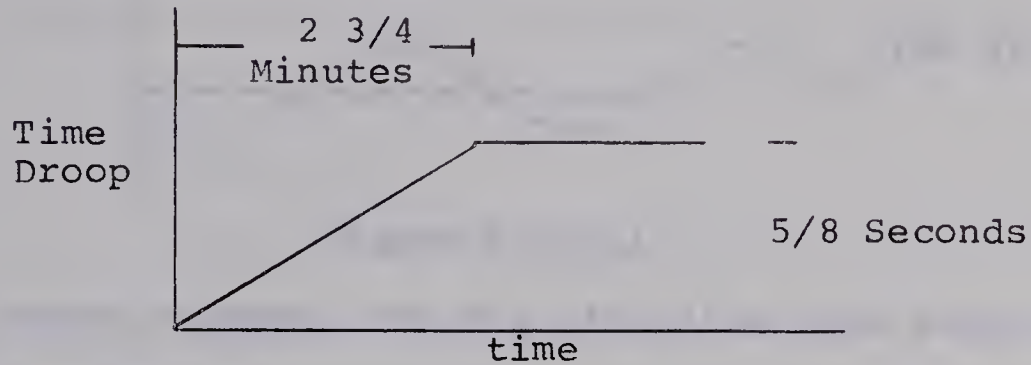


Figure II-5-9

The quadrant movement is represented by a time delay, with hard limiting applied after $t > T_0$ (refer to Figure II-5-10) where $T_0 = 2 \frac{3}{4}$ minutes.

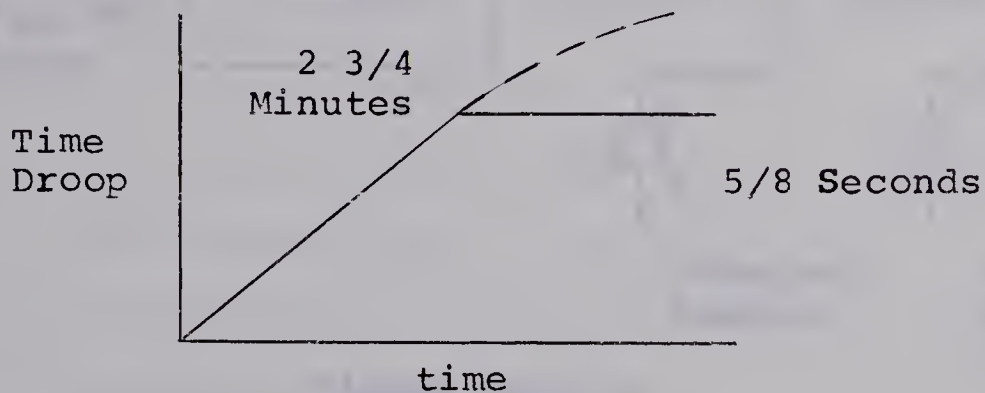


Figure II-5-10

The percentage load control is represented by increasing the magnitude of the generator power by the factor

$$\frac{100}{\% \text{ Load Setting}} \cdot$$

The Backlash Signal

The backlash signal is assumed to increase the time error signal as shown by Figure II-5-11.

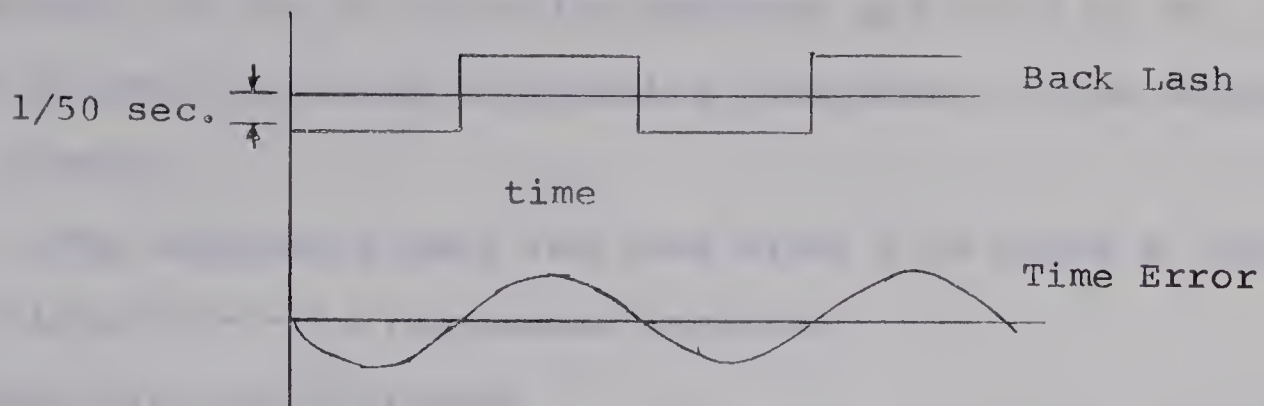


Figure II-5-11

The former accounts for the effective time error signal ($\Delta T'$) as seen by the controller. The operation of the controller on this signal is shown in Figure II-5-12.

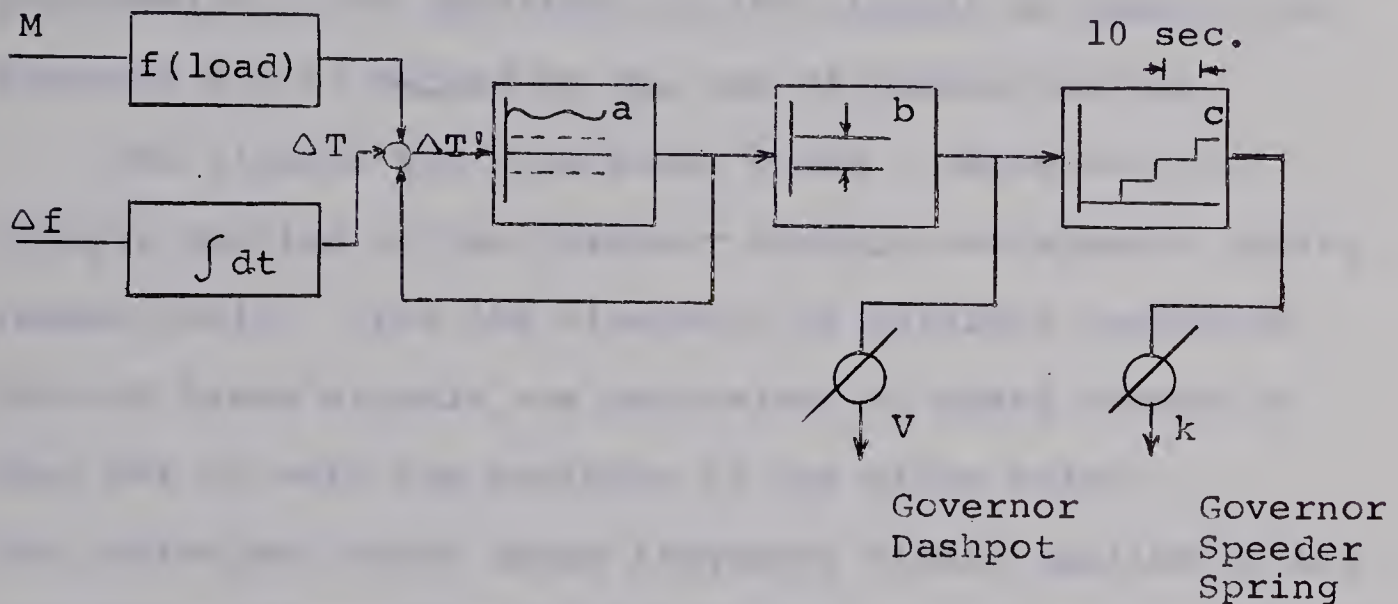


Figure II-5-12

The time error ($\Delta T'$) must be greater than the dead zone for the controller to respond. The input to block b is zero if $|\Delta T'| < \text{dead zone}$. The controller's output signals are of constant magnitude and not proportional to the time error. The time error signal is converted to one of constant magnitude in block b . The output signals of block b are sampled every ten seconds and accumulated in block c .

Comparators provide the required circuit components in blocks a and b . The sampling and accumulating functions

required the use of circuitry designed and built by Mr. J.A. Ash of the Electrical Engineering Department at the University of Alberta.

The feedback signal fed from block b to block a, shown in Figure II-5-11, represents backlash.

Controller Output Signals

It is to be noted that the magnitude of the controller signals are very carefully set and held constant. This is necessary as any error in the controller simulation is accumulated as the signals are summed in an arithmetic progression. The magnitude of the signals as seen by the governor may be varied by the use of potentiometers.

The signals fed from block b and c represent the signals applied to the governor dashpot and speeder spring respectively. From the viewpoint of governor operation, both of these signals are equivalent to speed errors as they act to vary the position of the pilot valve. The equivalent speed droop frequency signal applied by the controller is equal to $\Sigma k'B = \Sigma k$ (II-5-6)

The equivalent dashpot frequency signal applied by the controller (refer to Figure II-5-13).

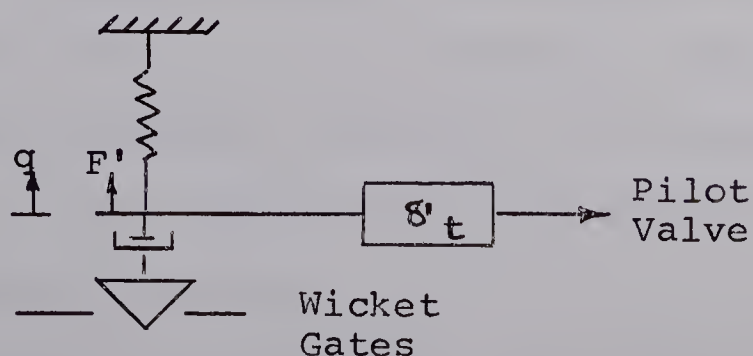


Figure II-5-13

Assuming that the wicket gate movement results from the pilot valve movement rather than from the applied dashpot force, the pilot valve movement resulting from this force is given by equation II-5-7.

$$v = q \delta'_t = \frac{\delta'_t}{C} \frac{F'}{1 + T_r s} = \frac{F}{1 + T_r s} \quad (\text{II-5-7})$$

Section II-5-4 Conclusion

The operation of the controller was found to be highly non-linear. As an accurate method of linearization was not evident, the non-linearities will be simulated.

CHAPTER III

Section III-0 The Analysis of Results

Section II-1-0 Introduction

The general observation of results accentuates two queries. The first being the oscillations superimposed on the frequency and synchronizing torque angle. The second is the large deviation in the time error when a step load change is applied. The cause of the forementioned oscillations are investigated and discussed in Section III-1-1. The time error resulting from a load change depends on the response of the controllers which in turn depends on the effective speed droop and dashpot frequency signals. The response of the controller, for an isolated generator and load, is discussed in Section III-1-2, from the viewpoint of minimizing the time error following a load change. At this stage, the dashpot frequency and time droop bias signals are grouped together and are defined as a damping signal, since they both act to decrease the time error seen by the controllers during the load change.

The optimum operation of the controllers directly depends on the settings of their parameters; the effective speed droop signals, the dashpot frequency signals, the time bias signals and the sampling duration. For the individual controller and generator, these parameters are interrelated among themselves and are also related to the permanent speed droop setting. Further, the parameter settings of the controllers are related from unit to unit through the permanent speed droop curves of the respective units.

To simplify the analysis, the relative optimum and then the optimum magnitude of the effective speed droop and damping

signals, assuming a sampling duration of 10 seconds, are investigated in Section III-1-3. By determining the optimum relative magnitude of the controllers' signals, the problem is reduced to finding the optimum magnitude of the signals for a single controller.

From the former analysis, the optimum magnitude of the effective speed droop signal is established, as related to the permanent speed droop setting and the percentage load setting of the individual unit. Further the optimum magnitude of the dashpot frequency signal is determined, as related to the effective speed droop signal.

The optimum value and effect of the sampling duration is determined in Section III-1-4, as related to the natural period of oscillation of the governing loop. The governing loop comprises the turbine, the generator and the governor. Following this, the effect of dead zone and backlash, which are inherent characteristics in any mechanical device, are discussed in Section III-1-5. A summation of the total analysis is then made in the conclusion, Section III-1-6.

Section III-1-1 The Sustained Synchronizing Torque Angle and System Frequency Oscillations

The sustained synchronizing torque angle and system frequency oscillations, shown in the computer results, will now be investigated. The generators and transmission lines supplying the load are assumed to be identical.

It is to be noted that these oscillations were not anticipated when the computer model was assembled and that, as the system damping was decreased, the oscillations

increased in magnitude to the point where the system became unstable. In an attempt to determine their source, the voltage regulator and controller models were isolated from the power system without noticeable effect. Therefore, these oscillations which have a natural frequency of approximately 2 c.p.s., must originate in the governing loop (Inner Loop) and/or the synchronizing torque loop (Outer Loop), shown in Figure III-1-1. As the former loop has a natural frequency of approximately 0.1 c.p.s., it is necessary that they originate with the latter.

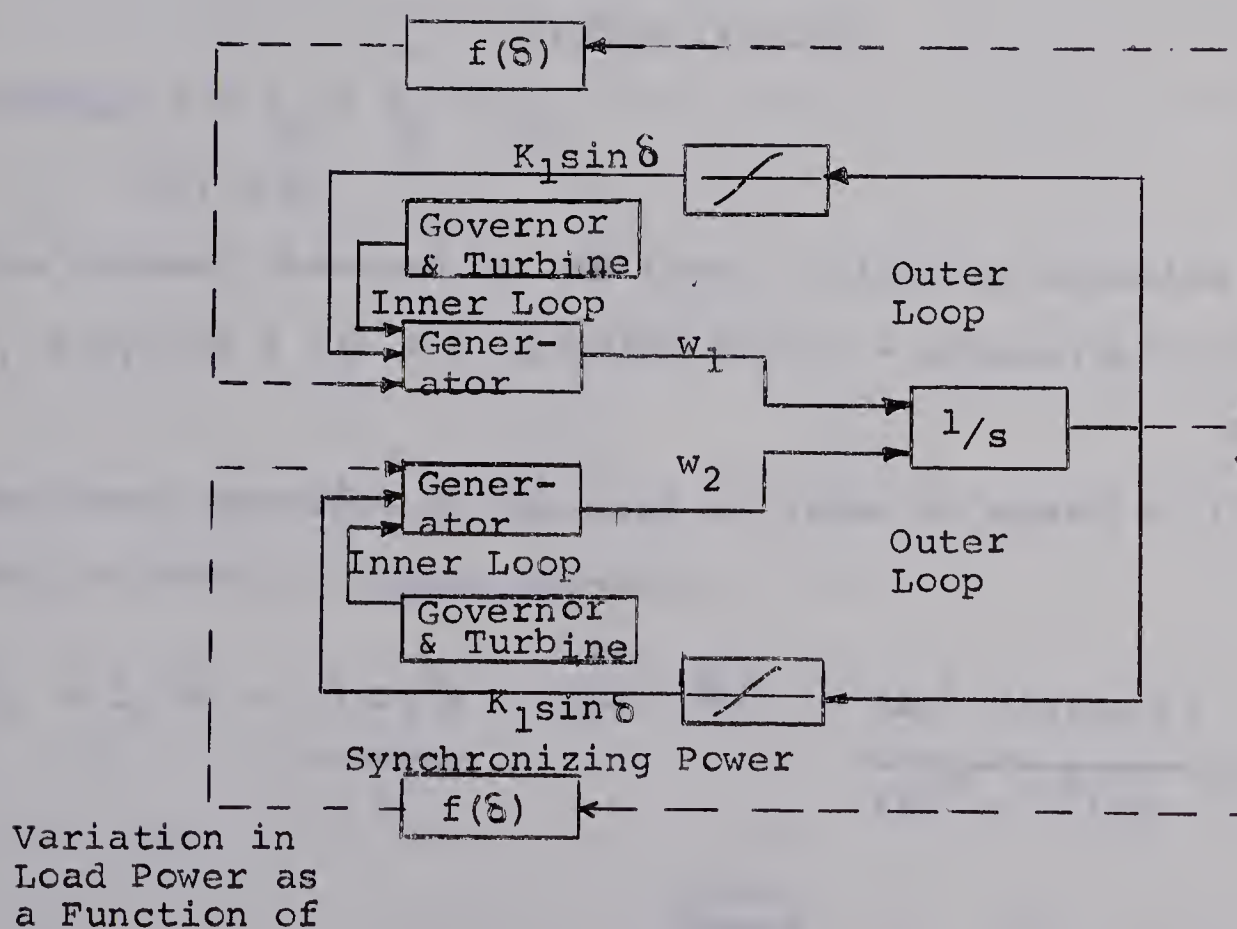


Figure III-1-1

For the system frequency oscillation to be caused by the outer loop, it is necessary that the power absorbed by the load vary as a function of the torque angle (δ) as

indicated by the dashed lines in Figure III-1-1. This will now be shown to be the case by assuming constant internal generator voltage (refer to Figure III-1-2.)

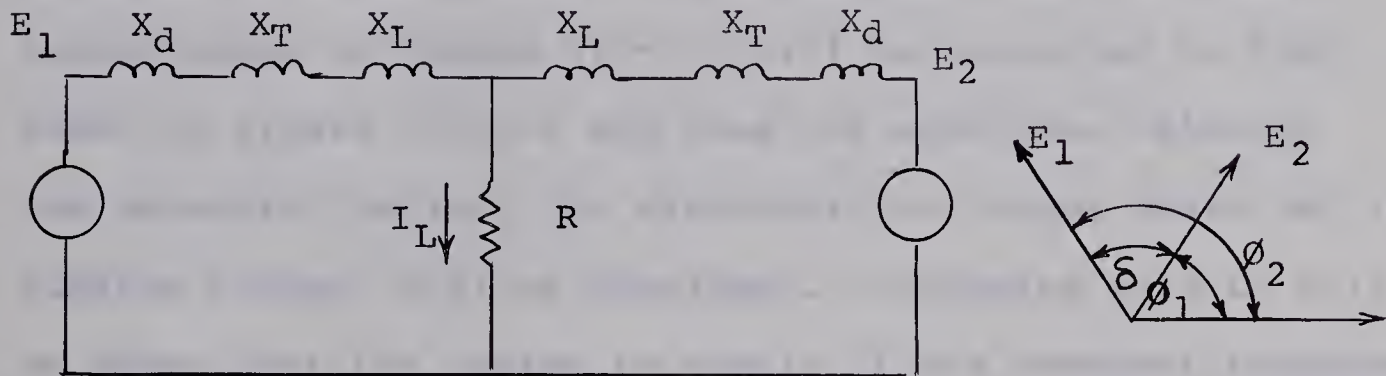


Figure III-1-2

Letting $X = X_d + X_T + X_L$

$$E_1 = E_2$$

The current absorbed by the load is given by equation III-1-1.

$$I_L = E_I / (2R + jX) + E_I \angle -\delta / (2R + jX) = 2E_I \cos(\delta/2) / (2R + jX) \quad (\text{III-1-1})$$

The power absorbed by the load is given by equation III-1-2 and is shown in Figure III-1-3.

$$P_L = I_L^2 R = \frac{4 E_1^2 R}{Z^2} \cos^2 \delta/2 = \frac{2 E_1^2 (1 + \cos \delta)}{(4R^2 + X^2)/R} \quad (\text{III-1-2})$$

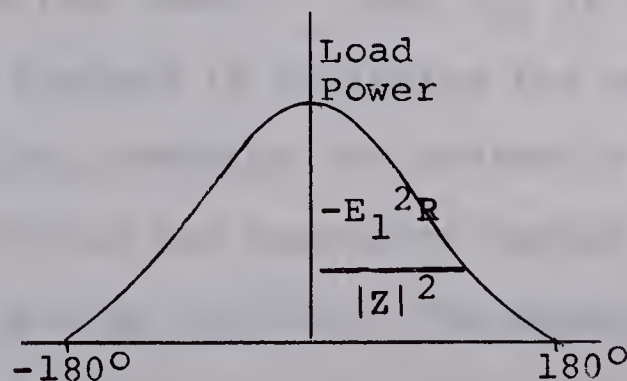


Figure III-1-3

From equation III-1-2 and Figure III-1-3, it is seen that an oscillation of the angle δ will produce an oscillation in the power absorbed by the load. The capability of this phenomena to produce oscillations in the system generators will now be investigated. To accomplish this the power system shown in Figure III-1-2 will be converted to that shown in Figure III-1-4 and then the equations relating the generator angles, the synchronizing torque angle and the turbine torque, will be developed. Following this it will be shown that the system is stable if the governor responses, inertia constants and damping constants are equal. Further, if the inertia constants are not equal and if the system damping lies within prescribed limits, it will be shown that sustained oscillations will result.

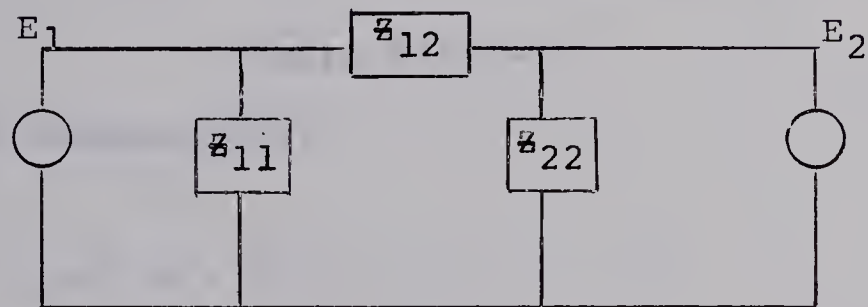


Figure III-1-4

As the generator voltages are assumed constant, the power supplied to the loads Z_{11} and Z_{22} is constant, and therefore may be ignored in analyzing the cause of the oscillations. Thus, assuming the system is in equilibrium, the equations relating the generator angles to the turbine and load torques are as follows^x. The equations are expressed in per unit quantities.

$$2H_1 \ddot{\delta}_1 + P_{d1} \dot{\delta}_1 = \Delta P_{T1} - \text{Generator Load Torque} \quad (\text{III-1-3})$$

$$2H_2 \ddot{\delta}_2 + P_{d2} \dot{\delta}_2 = \Delta P_{T2} - \text{Generator Load Torque} \quad (\text{III-1-4})$$

The variation in the generator load torque, as a function of the torque angle, will now be obtained (refer to Figure III-1-5). To obtain the formulae, the Superposition Theorem will be employed to obtain the currents I_1 and I_2 created by E_1 and E_2 acting alone.

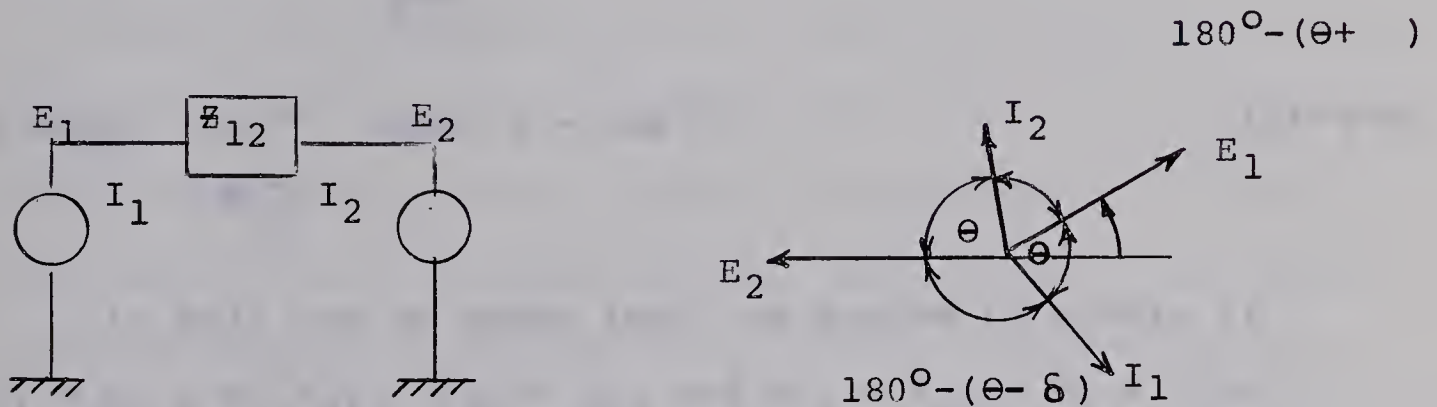


Figure III-1-5

Load Torque of Generator #1

$$\frac{E_1^2}{Z_{12}} \cos \theta - \cos (\theta + \delta) \quad (\text{III-1-5})$$

$$= \frac{E_1^2}{Z_{12}} \left[\cos \theta (1 - \cos \delta) + \sin \theta \sin \delta \right]$$

Load Torque of Generator #2

$$\frac{E_1^2}{Z_{12}} \left[\cos \theta - \cos (\theta - \delta) \right]$$

$$= \frac{E_1^2}{Z_{12}} \left[\cos \theta (1 - \cos \delta) + \sin \theta \sin (-\delta) \right] \quad (\text{III-1-6})$$

Thus equations III-1-7 and III-1-8 yield the relationship of the generator angles, the torque angles and the turbine torque.

$$2H_1 \ddot{\phi}_1 + \frac{P_{d1}}{E_1^2} \dot{\phi}_1 + \frac{1}{\overline{E}_{12}} \sin \theta \sin \delta$$

$$= \Delta P_{T1} - \frac{E_1^2}{\overline{E}_{12}} \cos \theta (1 - \cos \delta) \quad (\text{III-1-7})$$

$$2H_2 \ddot{\phi}_2 + \frac{P_{d2}}{E_1^2} \dot{\phi}_2 + \frac{1}{\overline{E}_{12}} \sin \theta \sin(-\delta)$$

$$= \Delta P_{T2} - \frac{E_1^2}{\overline{E}_{12}} \cos \theta (1 - \cos \delta) \quad (\text{III-1-8})$$

It will now be shown that the system is stable if $H_1 = H_2 = H$, $P_{d1} = P_{d2} = P_d$, and $P_{T1} = P_{T2} = P_T = f(w)$, by redefining $\phi_1 = \phi_0 + \delta/2$ and $\phi_2 = \phi_0 - \delta/2$.

As P_{T1} and P_{T2} are functions of w , the relationships are written as follows.

$$\Delta P_{T1} = f_1(\dot{\delta}, \dot{\phi}_0) = -F_1(\dot{\delta}) - G_1(\dot{\phi}_0) \quad (\text{III-1-9})$$

$$\Delta P_{T2} = f_2(\dot{\delta}, \dot{\phi}_0) = -F_2(\dot{\delta}) - G_2(\dot{\phi}_0) \quad (\text{III-1-10})$$

Now, as generator #2 sees the same angle ϕ_0 but the negative of the angle δ to that of generator #1, the relationships of F_1 to F_2 and G_1 to G_2 are

$$F_1(\dot{\delta}) = -F_2(\dot{\delta}) \quad (\text{III-1-11})$$

$$G_1(\dot{\phi}_0) = G_2(\dot{\phi}_0) \quad (\text{III-1-12})$$

Substituting the above relationships into equations III-1-7 and III-1-8, and after further manipulation equations III-1-13 and III-1-14 are obtained.

$$2H\ddot{\phi}_o + 2P_d\dot{\phi}_o + 2G(\phi_o) = \frac{2E_1^2}{Z_{12}} \cos\theta (1 - \cos\delta) \quad (\text{III-1-13})$$

$$H\ddot{\delta} + P_d\dot{\delta} + \frac{2E_1^2}{Z_{12}} \sin\theta \sin\delta + 2F(\dot{\delta}) = 0 \quad (\text{III-1-14})$$

From equations III-1-13 and III-1-14, it may be realized that the torque angle (δ) is independent of the datum angle (ϕ_o) and therefore the redefinition of ϕ_1 and ϕ_2 to $\phi_1 = \delta/2 + \phi_o$ and $\phi_2 = \delta/2 + \phi_o$, is valid.

From equation III-1-14 it is seen that any oscillation in the torque angle will damp out even with $P_d = 0$, as the governing loop provides damping through the term $2F(\dot{\delta})$. Thus the forcing function ($2E_1/Z_{12} \cos\theta \cos\delta$) of equation III-1-14 becomes a constant and therefore any oscillation in the system datum angle (ϕ_o) will be damped out through the governor response $G(\dot{\phi}_o)$.

Therefore, with equal governor responses, inertia and damping constants, the synchronizing torque and frequency oscillations, shown in the computer results, cannot be justified.

To determine the effect on the system when $H_1 \neq H_2$ in the absence of system damping, equation III-1-8 is subtracted from equation III-1-7 to obtain equation III-1-15.

$$2H_1\ddot{\phi}_1 - 2H_2\ddot{\phi}_2 = -\frac{2E_1^2}{Z_{12}} [\sin\theta \sin\delta] + \Delta P_{T1} - \Delta P_{T2}$$

Letting $\delta = \phi_1 - \phi_2$

$$\frac{2H_1 \ddot{\delta}}{E_{12}} + 2E_1^2 \sin\theta \sin\delta + \Delta P_{T2} - \Delta P_{T1} = 2(H_2 - H_1) \ddot{\phi}_2 \quad (\text{III-1-15})$$

Assuming a slight decrease in load, equations III-1-7 and III-1-8 show that both generators will accelerate. With H_2 greater than H_1 or visa versa, it is necessary for the generators to separate, to satisfy equation III-1-15. The right hand side of this equation cannot be zero when the units are accelerating. As the generators separate, the angle δ increases and consequently the load decreases. Since the governing loop response is slow with respect to that of the synchronizing torque loop, the load decreases more rapidly than the governor can cut back the input power from the turbine. As a result of this cumulative action, the system becomes unstable.

To make the system stable, the energy dissipated by system damping must be equal to or greater than the energy supplied to the system by a decreasing load. The energy dissipated by damping and that supplied by a decreasing load are given by equations III-1-16 and III-1-17 respectively. The energy dissipated by damping

$$\int P_d \frac{d\phi_1}{dt} d\phi_1 + \int P_d \frac{d\phi_2}{dt} d\phi_2$$

Letting $\phi_1 = \phi_2 + \delta$

$$P_d \left[2 \left[\frac{d\phi_2}{dt} \right]^2 + 2 \frac{d\phi_2}{dt} \frac{d\delta}{dt} + \left[\frac{d\delta}{dt} \right]^2 \right] dt \quad (\text{III-1-16})$$

The energy supplied by a decreasing load

$$\frac{E_1^2}{Z_{12}} \cos\theta (1 - \cos\delta) dt \quad (\text{III-1-17})$$

Now as oscillations in δ and ϕ_2 are found to be approximately sinusoidal in the computer results, the energy supplied to the system minus the energy dissipated is given by equation III-1-18. The angles δ and ϕ_2 are assumed to be equal to $A \sin(w_a t)$ and $B \sin(w_a t)$ respectively. The equation is expressed in per unit quantities with the magnitudes of the angles expressed in radians.

$$\int \frac{E_1^2}{Z_{12}} \left[\cos\theta \left[1 - \cos(A \sin w_a t) \right] - \frac{w_a^2 P_d}{(2\pi f_o)^2} \times \right. \\ \left. \left[2B^2 + 2AB + A^2 \right] \cos^2 w_a t \right] dt \quad (\text{III-1-18})$$

For small angle variations it may be seen from equation III-1-15 that $\phi_2 \approx C\delta$ as $\sin(\delta) \approx \delta$. Thus letting $\phi_2 = C\delta$, equation III-1-18 is rewritten below.

$$\int \left[\frac{E_1^2}{Z} \cos\theta \left[1 - \cos(A \sin w_a t) \right] - \frac{w_a^2 P_d}{(2\pi f_o)^2} \times \right. \\ \left. \left[2C^2 + C + 1 \right] A^2 \cos^2 w_a t \right] dt$$

From the coefficient of the time intergrand, it may be realized that, with $(2C^2 + C + 1)w_a^2 P_d$ greater than $E_1^2 \cos\theta$,

$$\frac{(2\pi f_o)^2}{Z_{12}}$$

the system will be stable and oscillations will not result. If the former is not the case however, it is not only unnecessary but also unlikely that the system will be unstable. This conjecture is supported by the following reasoning.

The system damping will attempt to hold the system together and in its efforts to accomplish this, it allows the governing loop more time to decrease the turbine power input to offset the decreased load. Further, as the energy made available by the decreasing load accelerates both units, provided H_1 or H_2 is not equal to infinity, it is unnecessary for the system damping to dissipate all the supplied energy to retain stability. Further, with increasing δ , the energy dissipated by damping increases more rapidly than the energy supplied to the system by decreasing load. This latter phenomena is believed to produce the sustained oscillations.

Based on the former equations, the average energy supplied to and dissipated by the decreased load and increased damping respectively are shown in Figures III-1-6 and III-1-7. As the average frequency is constant, the average energy supplied to and absorbed by the generator inertia is zero and therefore this energy is not considered.

The figures below do not incorporate the stabilizing action of the governing loop. However, it is felt that if the sustained oscillations will exist in the absence of this action, they can be made to exist with it, by decreasing the system damping (P_d). This reasoning is corroborated by equation III-1-19, which is obtained from the coefficient of time integrand of equation III-1-18.

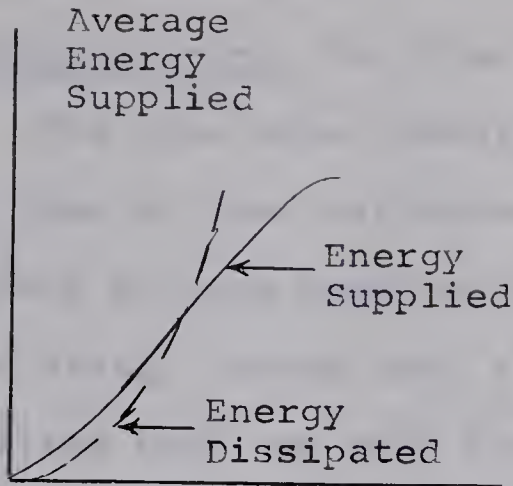


Figure III-1-6

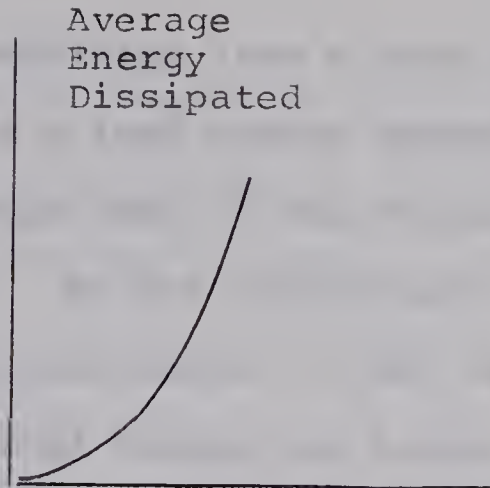


Figure III-1-7

Figures III-1-6 and III-1-7 show that, with

$$\frac{E_1^2 \cos \theta (2 f_o)^2}{\bar{E}_{12} (A_{\max} = 1.57)^2 w_a^2 (2C^2 + C + 1)} < P_d < \frac{E_1^2 \cos \theta (2 f_o)^2}{\bar{E}_{12} w_a^2 (2C^2 + C + 1)} \quad (\text{III-1-19})$$

the energy supplied to the system is greater than that dissipated for small variations in the angle δ and visa versa for large variations. This phenomena leads to sustained oscillations, since when the oscillations are small, they will increase to a given magnitude and when they are large they will decrease to the same given magnitude.

It is to be noted that the inertia constants were intended to be equal in the computer simulation. However, as the inertia constants directly affect the gain of the synchronizing torque loop, shown in Figure III-1-1, any error made in the potentiometer settings, affecting the gain of this loop, would produce the affect of unequal generator inertia constant.

In view of the above and on the basis of the former analysis, it is the contention of the author that these oscillations have a justified existence, and were caused by unequal

inertia constants.

Section III-1-2 The Time Error Resulting From a Load Change

The time error resulting from a load change depends on the type of load variation, the magnitude of the controllers' signals and the sampling duration. As the controllers operate on a static rather than a proportional basis, it may be realized that the more rapid the load change the larger will be the ensuing time error for a given magnitude of the controllers' parameters.

Assuming the power system comprises a single generator and load the optimum controller parameters, to minimize the time error, will now be determined for a step full load, a step quarter load and two ramp load changes. The first ramp load change will be equal to the maximum response of the controller (i.e. the controller is just able to hold the time error to a minimum value). The second load change will be slower than the maximum response of the unit.

In resume, as the system load and frequency are related through the permanent speed droop of the governor, a load change will produce a proportional steady-state frequency error. The controller integrates this frequency error and, acting to correct the resulting time error, applies two signals to the governor. The first is the effective speed droop signal which is permanently applied to correct the frequency error. The second is the dashpot frequency signal which is applied temporarily to correct the existing time error.

In addition to these signals, the load sharing feature

introduces a time bias signal in the controller, which allows the generators to lose time when they are picking up load and visa versa when they are dropping load. To the individual generator, each of which has its own controller, this signal acts very similar to the dashpot frequency signal. In its absence, it would be necessary to increase the dashpot frequency signal to prevent the units from losing this time. Thus, to simplify the analysis, the time droop and dashpot frequency signals are defined as a damping signal ($\dot{v}$). This damping signal is assumed to act in the same manner as the dashpot frequency signal. This approximation will be reconsidered when the optimum controller parameters are determined for a 10 second sampling period in Section III-1-3.

- To facilitate the forementioned investigation let
- K The frequency error equal to the permanent speed droop setting
 - k The effective speed droop signal
 - $\dot{v}$ the damping signal
 - m The sampling period
 - t_0 The minimum time required for the controller to raise the speed droop from its full-load to its no-load position

Step Full Load Change

It will now be shown that for critically damped time and frequency errors, the damping signal ($\dot{v}$) must be related to the effective speed droop signal by $\dot{v} = k t_0 / 2 m$. In making this analysis the response time of the governor, the turbine and the generator are assumed to be negligible in comparison to that of the controller.

The equations necessary to determine the controller parameters for a critically damped system, are developed as follows and are portrayed in Figure III-1-8.

Approximating the frequency error (Δf) as follows

$$\Delta f, \text{ for } 0 \leq t \leq m = K$$

$$\Delta f, \text{ for } t > m = K - \frac{k}{m} t$$

the time error is given by equation III-1-20

$$\Delta T = 1/60 \left[\int_0^m K + \int_m^t \left(K - \frac{k}{m} t \right) dt - \int_m^t \dot{v} dt \right]$$

$$\Delta T = 1/60 \left[Kt - \frac{kt^2}{2m} + \frac{km}{2} - \dot{v}t + \dot{v}m \right] \quad (\text{III-1-20})$$

To have a critically damped system it is necessary to have zero time error when the frequency error is nullified. The minimum time required for the controller to correct the frequency error (K) is $t = t_o + m$. Thus the required damping signal ($\dot{v}$) to yield a critically damped system, with a given effective speed droop signal (k), may be found by substituting $\Delta T = 0$ when $t = t_o + m$ in equation III-1-20.

$$0 = k(t_o + m) + \frac{k}{2m} - \frac{k}{2m} (t_o + m)^2 - \dot{v}(t_o)$$

$$\text{As } K = \frac{k}{m} t_o, \text{ the required damping signal is } \dot{v} = \frac{k t_o}{2m}$$

The time error resulting from this load change as well as; the effective speed droop signal, the damping signal and the frequency signals applied to the governor, are shown in Figure III-1-8.

The requirement for a large dashpot frequency signal

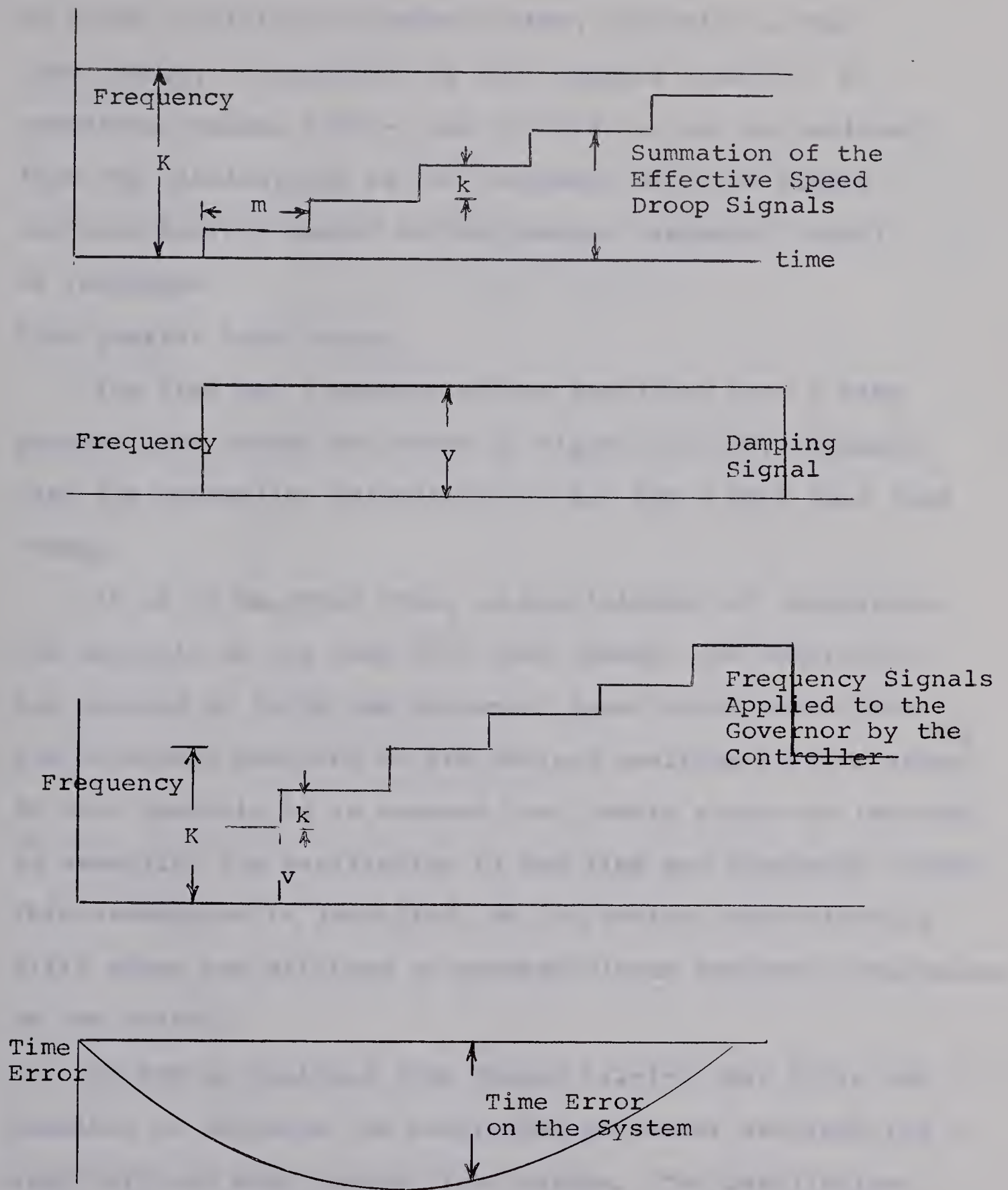


Figure III-1-8

to yield a critically damped system, following a step load change, is supported by the computer results. By comparing Figures III-2-6 and III-2-7, it may be realized that the oscillations in the frequency and time errors are more heavily damped as the dashpot frequency signal is increased.

Step Quarter Load Change

The time and frequency errors resulting from a step quarter load change are shown in Figure III-1-9, assuming that the controller parameters are set for a step full load change.

It is to be noted that, in the interest of simplifying the analysis of the step full load change, the controller was assumed to raise the permanent speed droop curve from its full-load position to its no-load position in five steps. In this analysis it is assumed that twenty steps are required to exemplify the oscillation in the time and frequency errors. This assumption is justified, as in practice approximately fifty steps are utilized to provide closer frequency tolerances on the system.

It may be realized from Figure III-1-9 that it is not possible to optimize the controller parameter settings for a step full and step quarter load change. The oscillations in the time and frequency errors, following the step quarter load change are intolerable. The optimum damping signal for a step quarter load change is obtained as follows.

Substitute $\Delta T = 0$ when $t = t_0/4 + m$ in equation III-1-20.

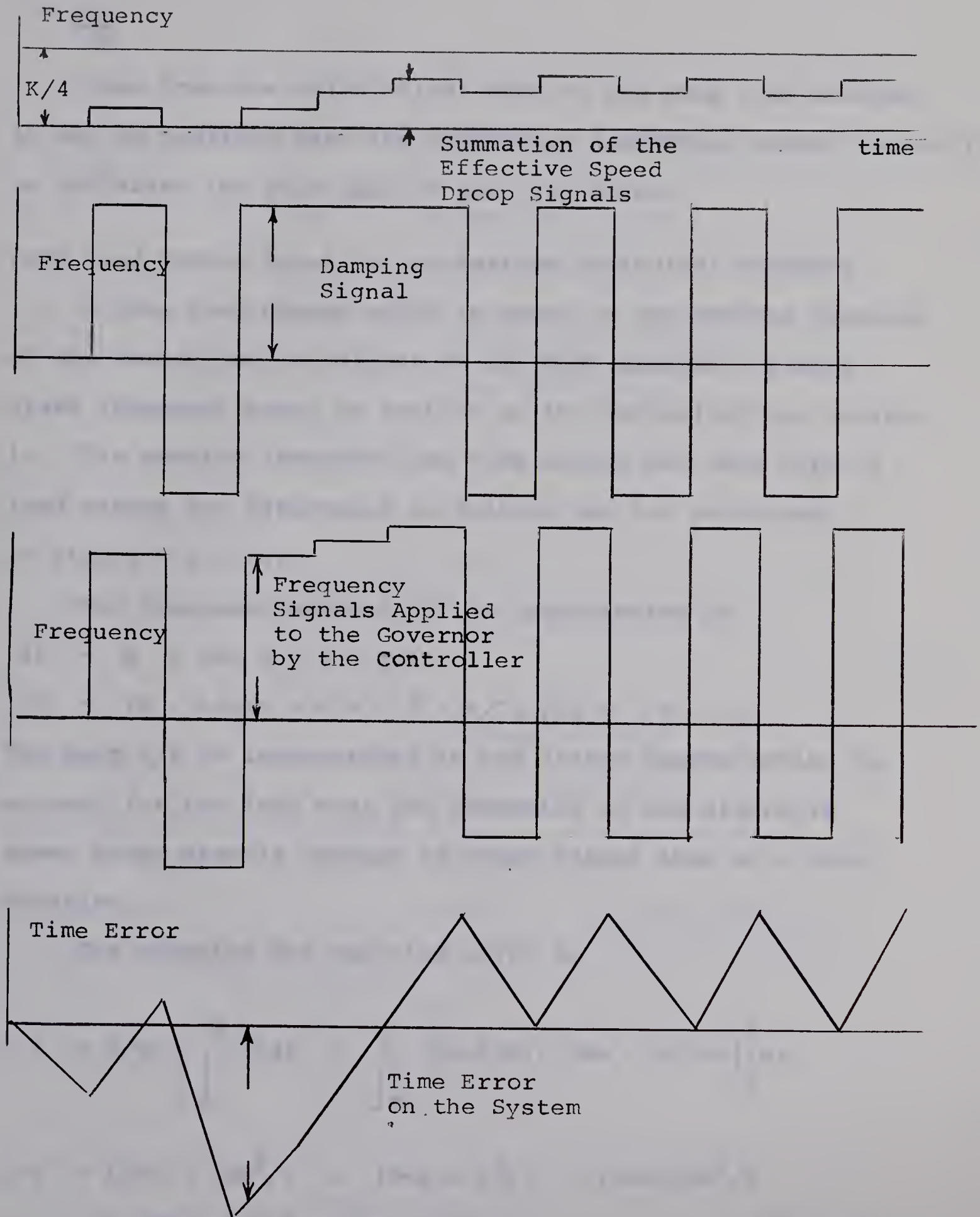


Figure III-1-9

$$0 = K/4 (t_o/4 + m) - \frac{k/m}{2} (t_o/4 + m)^2 + \frac{km}{2} - \dot{v} (t_o/4)$$

$$\dot{v} = \frac{k t_o}{8m}$$

Thus from the calculations made on the step load changes, it may be realized that the controller parameters cannot generally be optimized for this type of load variation.

Ramp Load Change Equal to the Maximum Controller Response

A ramp load change which is equal to the maximum response of the controller is defined as one that creates a steady state frequency error as rapidly as the controller can correct it. The ensuing frequency and time errors for this type of load change are determined as follows and are portrayed in Figure III-1-10.

The frequency error (Δf) is approximated by

$$\Delta f = N t \text{ for } 0 < t < m$$

$$\Delta f = (N - k/m)t + N m - \dot{v} - k/2, \text{ for } m < t < t_o$$

The term $k/2$ is incorporated in the latter approximation to account for the fact that the summation of the effective speed droop signals changes in steps rather than as a ramp function.

The equation for the time error is

$$\Delta T = 1/60 \left[\int_0^m N t dt + \int_m^t [(N - k/m)t + N m - k/2 - \dot{v}] dt \right]$$

$$\Delta T = 1/60 \left[N m^2/2 + (N - k/m) t^2/2 - (N - k/m) m^2/2 + (N m - \dot{v} - k/2) t - (N m - \dot{v} - k/2) m \right] \quad (\text{III-1-21})$$

As the frequency error produced by the load change is

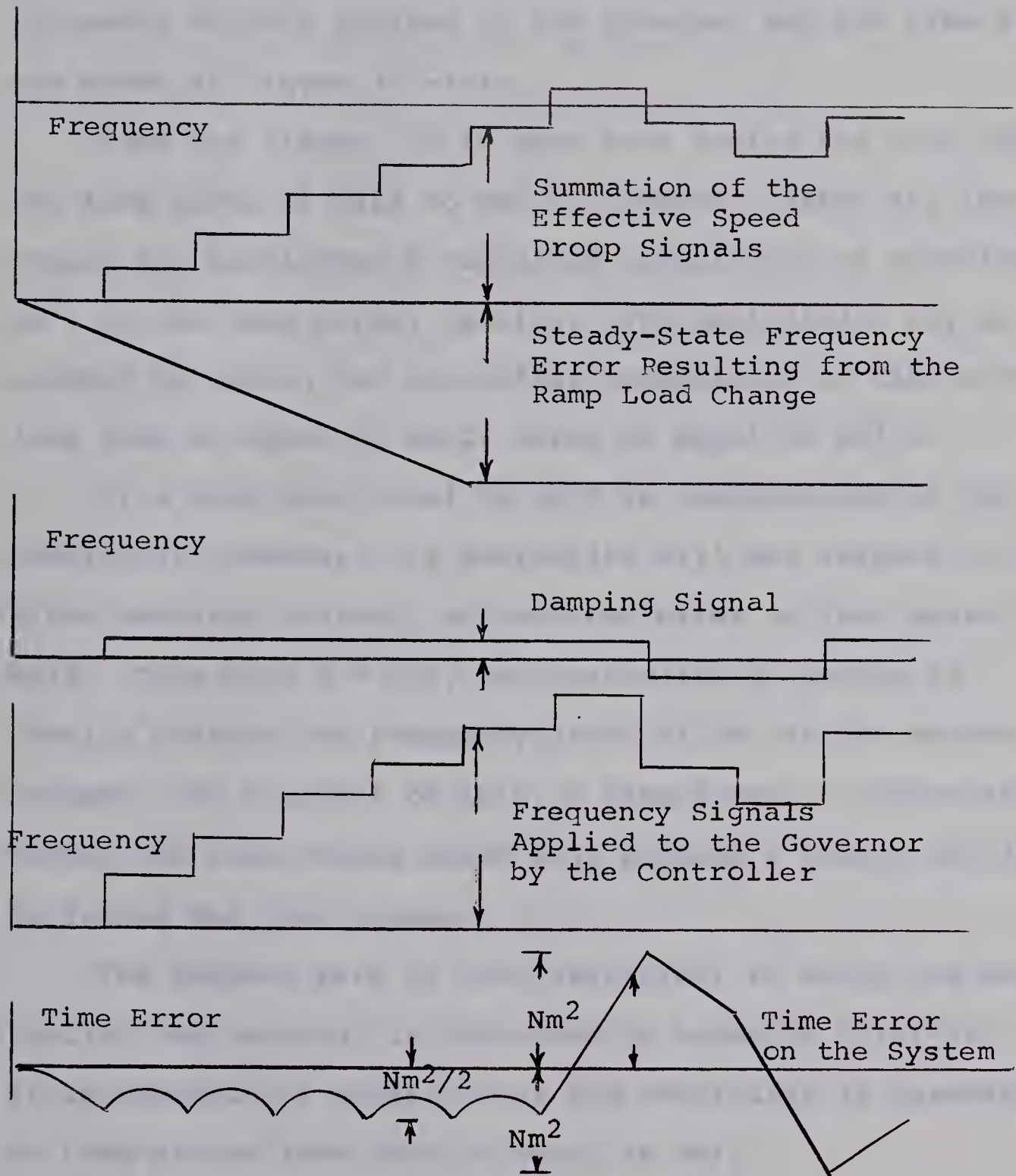


Figure III-1-10

equal to the maximum response of the controller ($N = k/m$), the time error may be held to a minimum by letting $\dot{v} = k/2$. Assuming $\dot{v} = k/2$; the effective speed droop signal, the frequency signals applied to the governor and the time error are shown in Figure III-1-10.

From the figure, it is seen that during the load change, the time error is held to $Nm^2/2$. However, after the load change has terminated a sustained oscillation of magnitude Nm^2 , in the time error, results. The oscillation may be removed by making the controller insensitive to time errors less than or equal to $km/2$, which is equal to $Nm^2/2$.

If a dead zone equal to $km/2$ is incorporated in the controller, however, the controller will not respond at the first sampling instant, as the time error is just equal to $km/2$. Thus with $\dot{v} = k/2$, the controller is unable to totally correct the frequency error of $2k$, at the second sampling instant. As a result of this, a time error is accumulated during the load change which will produce a damped oscillation following the load change.

The maximum rate of load variation, to which the controller can respond, is indicated by equation III-1-22. It is assumed, of course, that the controller is insensitive to time errors less than or equal to $km/2$.

Approximating the frequency error (Δf) by

$$\Delta f = Nt, \text{ for } 0 < t < 2m$$

$$\Delta f = 2Nm + \left[N - k/m \right] t - \dot{v} - k/2, \text{ for } 2m < t < t_1$$

where

t_1 is defined as the time when $\Delta T \leq km/2$ or $t_1 \geq t_0$, which-

ever occurs first. It is necessary to limit the frequency approximation in this manner as, if $\Delta T \leq km/2$ at a sampling instant, the summation of the effective speed droop signal does not increase and the equation becomes invalid.

The time error is given by equation III-1-22.

$$\begin{aligned} \Delta T &= 1/60 \left[\int_0^{2m} N t dt + \int_{2m}^{t_1} \left[(N-k/m)t + 2Nm - \dot{v} - k/2 \right] dt \right] \\ \Delta T &= 1/60 \left[2(N-k/m)m^2 + (\dot{v} + k/2)m - 2Nm^2 \right. \\ &\quad \left. + (N-k/m)t_1^2 / 2 + (2Nm - \dot{v} - k/2)t_1 \right] \quad (\text{III-1-22}) \end{aligned}$$

Assuming a load change $N = k/m$, with $\dot{v} = k/2$, it may be realized that the time error will accumulate until the load change terminates. The controller will then produce a damped oscillation in the frequency and time error, as it must over regulate to correct the residual time error. Thus it is necessary to either suffer these oscillations, for load changes more rapid than $N = k/2m$, or increase the dead zone to km and the damping signal to $3/2$ the effective speed droop signal. If the latter method of operation were to be implemented, it would be necessary to accept a system frequency error equal to the average value of the dashpot frequency signal. Otherwise a time error would not exist to activate the controller.

The operation of the controller for a ramp load change ($N = k/2m$, with $\dot{v} = k/2$) is shown in Figure III-1-11. From the dashed lines through the controller's response, it may be realized that the controller is capable of correcting the frequency error created by the load change. Further,

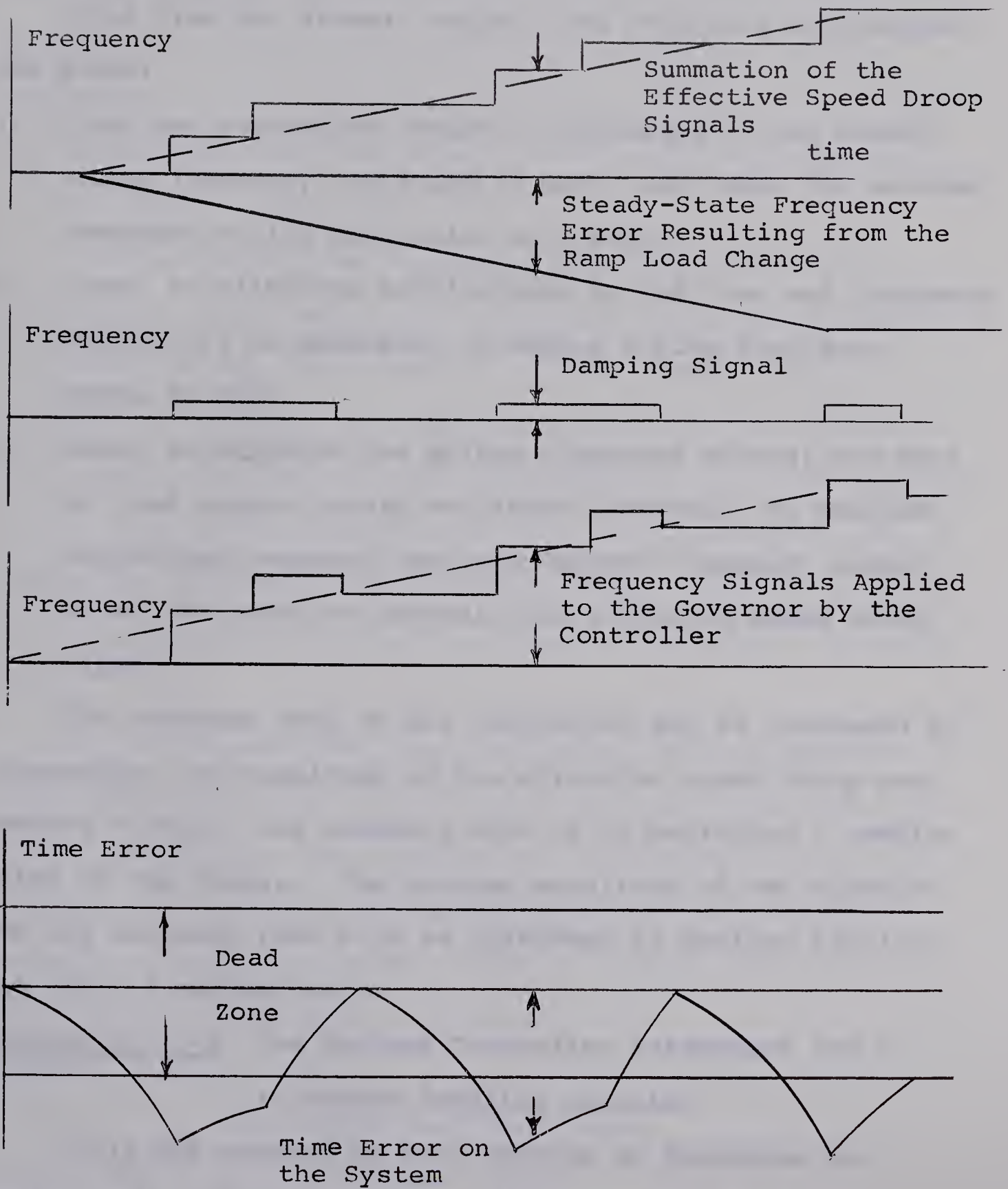


Figure III-1-11

as the controller is able to adjust its rate of response for slower load variations, it is able to minimize the time and frequency errors for load changes less rapid than $N = k/2m$.

Thus from the former analysis the following conclusions are drawn:

- 1) that the controller cannot be optimized if the steady state frequency variation is more rapid than the maximum response of the controller ($N = k/m$).
- 2) that, to eliminate oscillations in the time and frequency errors, it is necessary to employ a time dead zone equal to $km/2$.
- 3) that, to minimize the system frequency errors; the rate of load change should not exceed one-half the maximum controller response, and the dashpot frequency signal should be equal to one-half the effective speed droop signal.

The response rate of the controller may be increased by increasing; the magnitude of the effective speed droop and damping signals, the sampling rate or by employing a combination of the former. The optimum magnitude of the signals and the sampling rate will be discussed in Section III-1-3 and III-1-4 respectively.

Section III-1-3 The Optimum Controller Parameters for a 10 Second Sampling Duration

It is the purpose of this section to determine the optimum value of the controllers' parameters for a 10 second sampling duration. These parameters will first be determined theoretically and then will be verified by the

computer results.

The optimum relative magnitude of the signals, for controllers on different generators, will be investigated first and then the optimum magnitude of the signals, for the controller on the individual generator, will be determined. Following this, the theoretical results will be compared to the computer results.

For optimum system operation, the load should be correctly divided among the generating units with minimum operation of the controllers. As each generator has a separate controller, the optimum magnitude of the signals will vary from unit to unit, depending on the characteristics of the respective generating units. It is expected that the larger the permanent speed droop setting, the larger should be the controller's signals to correct the system frequency error.

It is assumed that the governing loop response is rapid with respect to that of the controller and therefore may be neglected. This subject is investigated in Section III-1-4. To determine the optimum relative controller signals, a system of n generators is assumed and the equations are developed as follows:

Define

Δf	the system frequency change
$\Delta M_T = Nt$	the system load change
$\Delta M_1, \dots, M_n$	the load change carried by each unit
$S_{f1}, \dots, S_{fn}$	the rate of frequency change as related

to the load change

$\gamma_1, \dots, \gamma_n$ the percentage load setting of each generator

$S_{11}t, \dots, S_{1n}t$ the response of each controller, comprising the summation of the effective speed droop signals and the damping signal

The system frequency change will be the same for all generators, therefore

$$\Delta f = S_{f1} \Delta M_1 - S_{11}t = \dots = S_{fn} \Delta M_n - S_{1n}t \quad (\text{III-1-23})$$

As the system load change must be carried by the generators

$$\Delta M_T = \sum_1^n \Delta M_n \quad (\text{III-1-24})$$

It is desired that each generator will absorb a given percentage of the total load change. Thus, to determine the sufficient conditions for this to be accomplished, assume that each unit absorbs a percentage of some load ΔM .

where

$$\Delta M_1, \dots, \Delta M_n = \gamma_1 \Delta M, \dots, \gamma_n \Delta M$$

Therefore

$$\Delta M = \frac{\Delta M_T}{\sum_1^n \gamma_n} = \frac{Nt}{\sum_1^n \gamma_n}$$

$$\Delta M_1, \dots, \Delta M_n = \frac{\gamma_1 Nt}{\sum_1^n \gamma_n}, \dots, \frac{\gamma_n Nt}{\sum_1^n \gamma_n}$$

(III-1-25)

Substituting the desired load apportionments into equation III-1-23, the controller responses, sufficient to achieve these load divisions, are determined by equation III-1-27. This equation is developed through equation III-1-26.

$$\frac{S_{f1} \gamma_1^{Nt} - S_{11}^t}{\sum_1^n \gamma_n} = , \dots , = \frac{S_{fn} \gamma_n^{Nt} - S_{1n}^t}{\sum_1^n \gamma_n}$$

Utilizing generator #1 as the reference

$$\frac{N}{\sum_1^n \gamma_n} [S_{f1} \gamma_1 - S_{fn} \gamma_n] = [S_{11} - S_{1n}] \quad (\text{III-1-26})$$

As the controller is able to vary its response for load changes slower than a given maximum, it is only necessary to consider the ratio of the maximum controller responses, to the permanent speed droop curves and percentage load settings, to determine the relative optimum controller parameters. Equation III-1-27, shows that the signals' magnitude should vary directly with the permanent speed droop curve and percentage load settings. Further, these settings are independent of the size of the system.

$$S_{1n} = \frac{\gamma_n S_{fn}}{\gamma_1 S_{f1}} S_{11} \quad (\text{III-1-27})$$

This hypothesis will now be corroborated by the computer results. With the generators on 100 per cent load control, the magnitude of the effective speed droop signals, as related to the permanent speed droop settings of the governor, will now be considered. When the effective speed droop signals are proportional to the permanent speed droop settings of the governor, it may be seen, as shown in Figure III-2-1, that following a step load change the load is correctly divided between the generators. However, when the former is not the case, the load is not correctly divided

following a step load change. Thus, as may be seen from Figures III-2-2 and III-2-3, it is necessary for the load sharing feature to redisturb the load. As a result, the system is subjected to unnecessary power disturbances.

The effect of the load sharing feature on the optimum settings of the effective speed droop signals may be seen from Figures III-2-4 and III-2-5. Figure II-2-4 indicates the power disturbances incurred on the system, following a step load change, when the units are on 100 and 60 per cent load control, and the effective speed droop signals are set for 100 per cent load control. It may be seen that, following the load disturbance, it is again necessary for the load sharing feature to apportion the load between the generators. Figure III-2-5 shows that if the magnitude of the effective speed droop signals are reduced as the percentage load setting is reduced, the units will pick up their correct share of the load.

It is concluded therefore, on the basis of the former analysis and the computer results, that equation III-1-27 provides a criterion to determine optimum relative magnitude of the effective speed droop signals. If the controller's parameters are set in accordance with this criterion, the controllers will respond as one unit. Thus when the optimum magnitude of the signals for the base or first unit is determined, the optimum magnitude of the signals for the other units is known.

The effect of other than optimum relative controller parameters will now be considered. It will be first assumed

that all generators are on 100 per cent load control, and that the effective speed droop signals, expressed as a percentage of the permanent speed droop settings, are not constant. As the controller's are capable of adjusting their response rates, for load changes less rapid than their maximum response, the rate of their responses will be equal for load variations less rapid than one-half the maximum response of the slowest unit. Thus for the above case, the relative magnitude of the controller signals is not significant. Therefore, only load variations in excess of the forementioned value will be considered. Assuming that the controller response of one unit is one-half that of the other (or others) and that a load change occurs which is equal to the maximum response of the slower unit, this unit will pick up its share of the load but will not do its share in correcting the frequency error accumulated before the time error exceeds the dead zone. Thus the maximum rate of load change is limited, by the controller's ability to minimize the time and frequency errors, to one-half the average value of the controller responses. If the response rate of any unit is less than one-half that of the other unit (or units), the maximum rate of load change is limited by the controller's ability to have the respective generators carry their correct share of the load, or by the former criteria, whichever is the lesser. Assuming the former limitation dictates and that the load change exceeds the maximum response of the slower unit, this unit will not absorb its correct portion of the load during the load change. Consequently a disturbance is

applied to the system at the termination of the load change, when the load sharing feature re-distributes the load.

It will now be assumed that all of the generators are not on 100 per cent load control. It is to be noted that the load sharing feature is able to apply the full time droop bias signal (5/8 seconds), as a ramp function, in 165 seconds. Also note that the lower the percentage load setting on a particular unit, the lower is the units required response since it assumes a smaller portion of the load. Thus, if the per unit load variation, which any unit is required to absorb, does not occur in less than 165 seconds, little advantage is to be gained by reducing the controller signals in accordance with the percentage load settings.

Therefore, although equation III-1-27 provides the criteria to determine the optimum relative magnitude of the controller signals, it is the opinion of the author that it is unnecessary to vary the magnitude of the controller signals in accordance with the percentage load settings. It will be shown in Section III-1-4 that the optimum maximum controller response is 500 seconds. Therefore, on account of the ensuing frequency oscillations, it is highly undesirable to have a per unit load change on any unit, occur in 165 seconds. The rate of per unit load change on the generators may be decreased by placing more units on the system, under the regulation of the controllers. In conclusion, maximum performance may be obtained if the relative controller signals, expressed as a percentage of the permanent speed droop settings, are constant.

The optimum magnitude of the effective speed droop signals are determined by the characteristics of the power system rather than by that of the controller. A limit is placed on the magnitude of the effective speed droop signal, as the larger its magnitude, the larger the power disturbance created when the controller adjusts the permanent speed droop curve. If only one station adjusts its speed droop curve at a given instant, it will assume a higher percentage of the system load. Consequently a power disturbance is fed to the other stations. The magnitude of this disturbance is equal to the sum of the effective speed droop and dashpot frequency signals, expressed as a percentage of the permanent speed droop setting.

The larger the magnitude of the effective speed droop signal, however, the more rapid is the response of the controller, as may be seen by comparing Figures III-2-7 and III-2-8. Consequently, to handle rapid system load variations, large effective speed droop signals are desired.

As the power oscillations of the generator stations vary between 1 and 5 per cent of the station capacity, it is felt that the disturbances imposed by the controller should be limited to 2 or 3 per cent of the station capacity. To accomplish this the magnitude of the effective speed droop signal must be limited to 2 per cent of the permanent speed droop setting, assuming a dashpot frequency signal equal to one-half the effective speed droop signal. It will be shown subsequently that this is the optimum magnitude of the latter signal.

Thus, in the interest of minimizing the power disturbances and in maximizing the controller response, an effective speed droop signal equal to 2 per cent of the permanent speed droop setting is considered to be optimum.

Accepting the above as the optimum effective speed droop signal, the optimum dashpot frequency signal will now be determined. It was found in Section III-1-2 that the magnitude of the damping signal, which comprises the dashpot and time droop basis signals, should be equal to one-half the magnitude of the effective speed signals, if the system frequency and time errors were to be minimized. Thus to obtain the optimum magnitude of the dashpot frequency signal, it is necessary to determine the contribution of the time droop bias signal to the damping signal.

In resume, when a load variation occurs, the Kelvin Balance activates a reversing motor to vary the quadrant's position according to the load absorbed by the respective generator. If the load variation is very rapid the mechanism is unable to follow, as it takes 165 seconds for the quadrant to travel from its no-load to its full-load position. However, for per unit ramp load, or slower load changes, occurring on the individual generator in 165 seconds or less, the load sharing feature is able to respond immediately. Thus the damping provided by the time droop bias signal, in c.p.s., is given by equation III-1-28.

$$U = \frac{y (5/8) (60)}{\delta t_1}$$

(III-1-28)

where

U damping signal provided by the time droop basis signal
in c.p.s.

y per unit load change of the individual generator
capacity ($\Delta M_n / M_{nr}$)

δ percentage load setting of the generator

t_1 time required for the load to change

It may be realized from equation III-1-28, that the damping, provided by the time droop signal decreases with slower and smaller load changes. It would appear that the damping would increase as the percentage load setting increases. This is not the case, however, as the load change absorbed by this unit decreases.

The average damping provided by the dashpot frequency signal also decreases with slower and small load changes. For a ramp load change, equal to one-half the maximum response of the controller, this signal is applied only during one-half the time interval of the load change as may be seen from Figure III-1-11. Thus the average damping provided by the dashpot frequency signal is given by equation III-1-29, in c.p.s.

$$\text{Average Dashpot Freq. Sig.} = \frac{y t_o}{t_1} \quad V \quad (\text{III-1-29})$$

where

t_o the minimum time required for the controller to raise
the speed droop curve from its no-load to its full-load
position ($\frac{\delta_p}{k} = 500 \text{ seconds}$)

The damping signal, determined in Section III-1-2, was assumed to have the same shape as the dashpot frequency signal. Therefore the average damping signal, required to minimize the system frequency error, is equal to $y(t_0/t_1) (k/2)$. The required dashpot frequency signal, to yield this value of damping during the load change, is obtained by equating the former values and is given by equation III-1-30.

$$V = \frac{\delta_p}{100} - \frac{(5/8)(60)}{500}$$

$$V = \frac{\delta_p}{100} - 0.075 \quad (\text{III-1-30})$$

To minimize the system frequency error it would appear from equation III-1-30, that with a 2 or 4 per cent permanent speed droop setting, the dashpot frequency signal should be made negative. However, when one reconsiders the controllers' operation, it may be seen that this is not the case.

The signal actuating the controller is the actual time error minus the time droop signal. Therefore, for a ramp load change equal to one-half the maximum response of the controller, the frequency error must equal and remain at 0.0375 c.p.s. Otherwise a time error will not exist to actuate the controller. If the dashpot frequency signal is made negative, it will act to increase the frequency error and will thereby provide an activating time error. The controller will respond to reduce the frequency error to 0.0375 c.p.s. and, as the dashpot frequency signal is removed when the load change terminates, a frequency error

will not remain on the system. It is assumed, of course, that the time error is equal to zero when the load change terminates. On the other hand, if the dashpot frequency signal is made positive, a frequency error of 0.0375 c.p.s. will remain, at the termination of the load variation.

It is to be noted that in either of the above cases, a step frequency error of 0.0375 c.p.s. is applied to the controller. Therefore, no advantage is to be gained through a negative dashpot frequency signal. Further, if this signal is made negative and a time error should be incurred when the load is constant, the system will become unstable as the controller is negatively damped. Therefore the dashpot frequency signal should be positive.

The criteria for establishing the magnitude of the dashpot frequency signal are; a large signal is desired to maximize the damping, a small signal is desired to minimize the system frequency error and power disturbances. The latter criteria governs as a large damping signal is not required, provided the load change does not exceed one-half the maximum controller response. Further, the magnitude of the system frequency error, created by this signal is not large. In the interest of providing some damping and in minimizing the power disturbances to 3 per cent of the generator capacity, it is concluded that the dashpot frequency signal should equal one-half the effective speed droop signal. It is possible to increase the dashpot frequency signal and decrease the effective speed droop signal without adversely affecting the power disturbances, however, this would degrade

the maximum controller response.

It would appear possible to incorporate another signal to counteract the frequency error incurred through the time droop bias signal. However, if this signal were to be implemented, it would cause the generators to drop load when the load is increasing and visa versa. As this effect is undesirable, it must be weighted against the effect of a step frequency disturbance in the controller.

The step frequency error created by the time droop bias signal does not significantly degrade the operation of the controller. For a load change equal to one-half the maximum response of the controller, this frequency error is equal to 3 and 1 1/2 per cent of a 2 and 4 per cent permanent speed droop setting respectively. As this disturbance does not exceed that imposed by the controller signals it is considered acceptable.

In summary, it is concluded that the effective speed droop and dashpot frequency signals should be equal to 2 and 1 per cent of the permanent speed droop setting, when the units are on 100 per cent load control. It was found unnecessary, though possibly preferable, to reduce the magnitude of the controller signals as the per cent load setting is reduced.

Section III-1-4 The Optimum Sampling Period

The response rate of the controllers, for a given magnitude of the effective speed droop signals, may be increased by decreasing the sampling period. It will now be shown theoretically, however, that the maximum

sampling rate (minimum sampling period is limited by the transient response of the governing loop).

The controller response comprises the load sharing feature as well as the dashpot frequency and effective speed droop signals. The load sharing feature will be considered first, although it is not altered by the sampling period. It will then be determined if this feature has a detrimental affect on the governing loop response. Following this, the effect of the dashpot frequency and effective speed droop signals will be considered. The rate of summation of the effective speed droop signals is directly related to the sampling rate.

To determine the effect of the controller, on the transient response of the governing loop, an impulse load change is applied. The governing loop and controller responses are then taken individually. By comparing these responses, the effect of the controller on the governing loop is determined. The results of this analysis will then be compared to the computer results.

The load sharing feature is effected through the quadrant movement, by the signal ΔT_D , as shown in Figure III-1-12

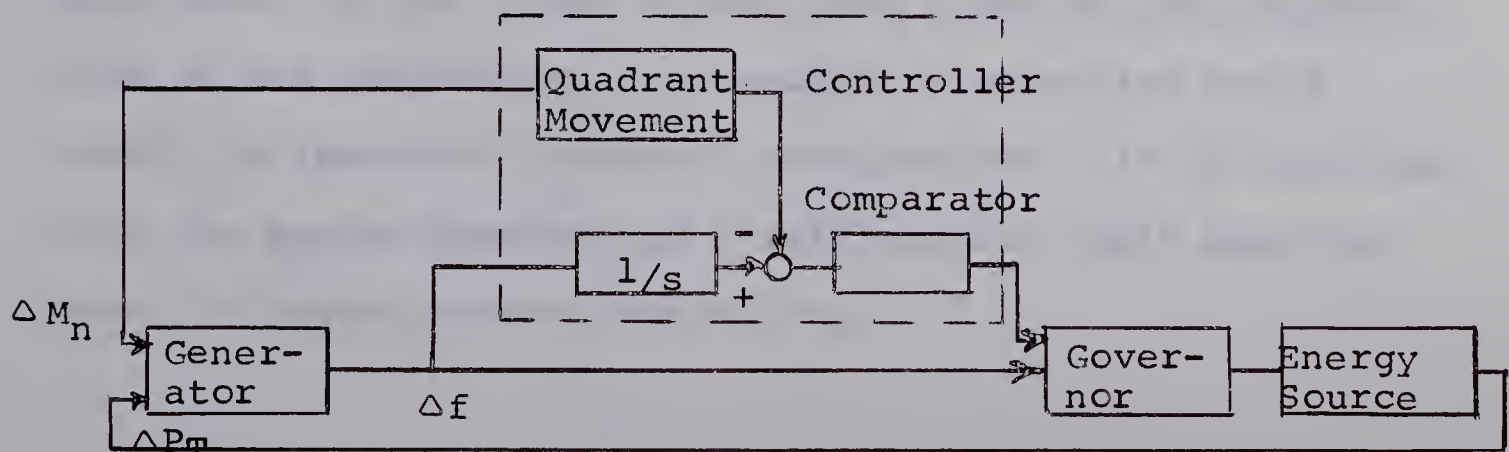


Figure III-1-12

The relationship of the quadrant movement to the generator output power, for rapidly varying loads, is approximately given by equation III-1-31.

$$\Delta T = \frac{\Delta M_n / M_{nr}}{1 + T_d s} \quad (\text{III-1-31})$$

As the time droop signal subtracts from the actual time error, the load sharing feature reduces the time error which activates the controller. The relationship of $\Delta M_n / M_{nr}$, ΔT and ΔT_D are shown in Figure III-1-13.

From observation of Figure III-1-13, it is concluded that the load sharing feature does not degrade the transient response of the governing loop. This may be realized intuitively as the load sharing feature reduces the commanding signal, through which the controller has the potential of degrading the system performance.

The effective speed droop and dashpot frequency signals are shown in Figure III-1-14, for the same impulse load change. A rapid sampling rate or a short sampling period is assumed.

From Figure III-1-14, it may be realized that the summation of the effective speed droop signals is in phase with the frequency error existing in the governing loop. As the magnitudes of the former signals are fixed by the response rate of the controller, this method of operation would result in sustained frequency oscillations. It is possible, with low system damping and a sufficiently rapid sampling rate, to create system instability.

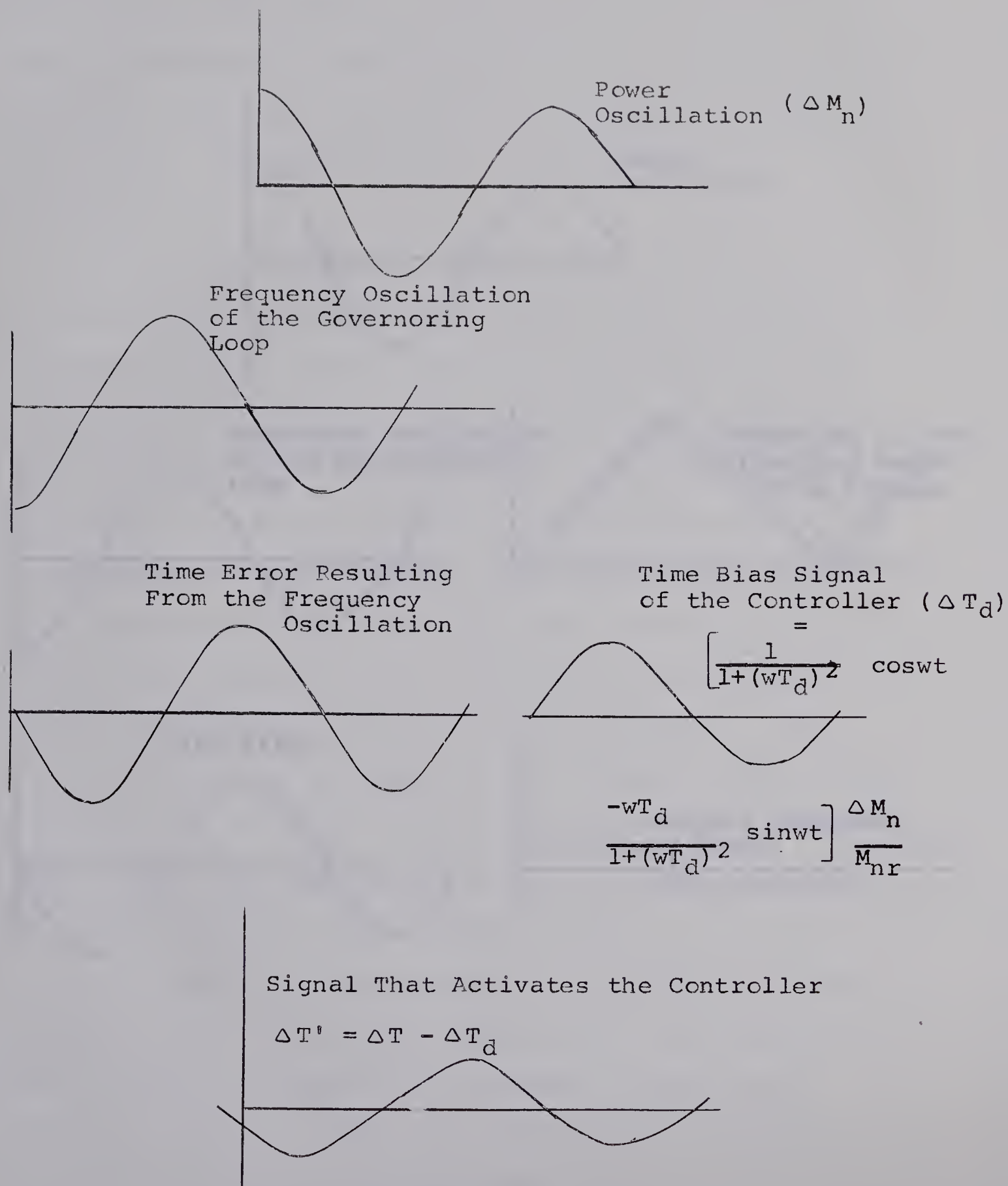


Figure III-1-13

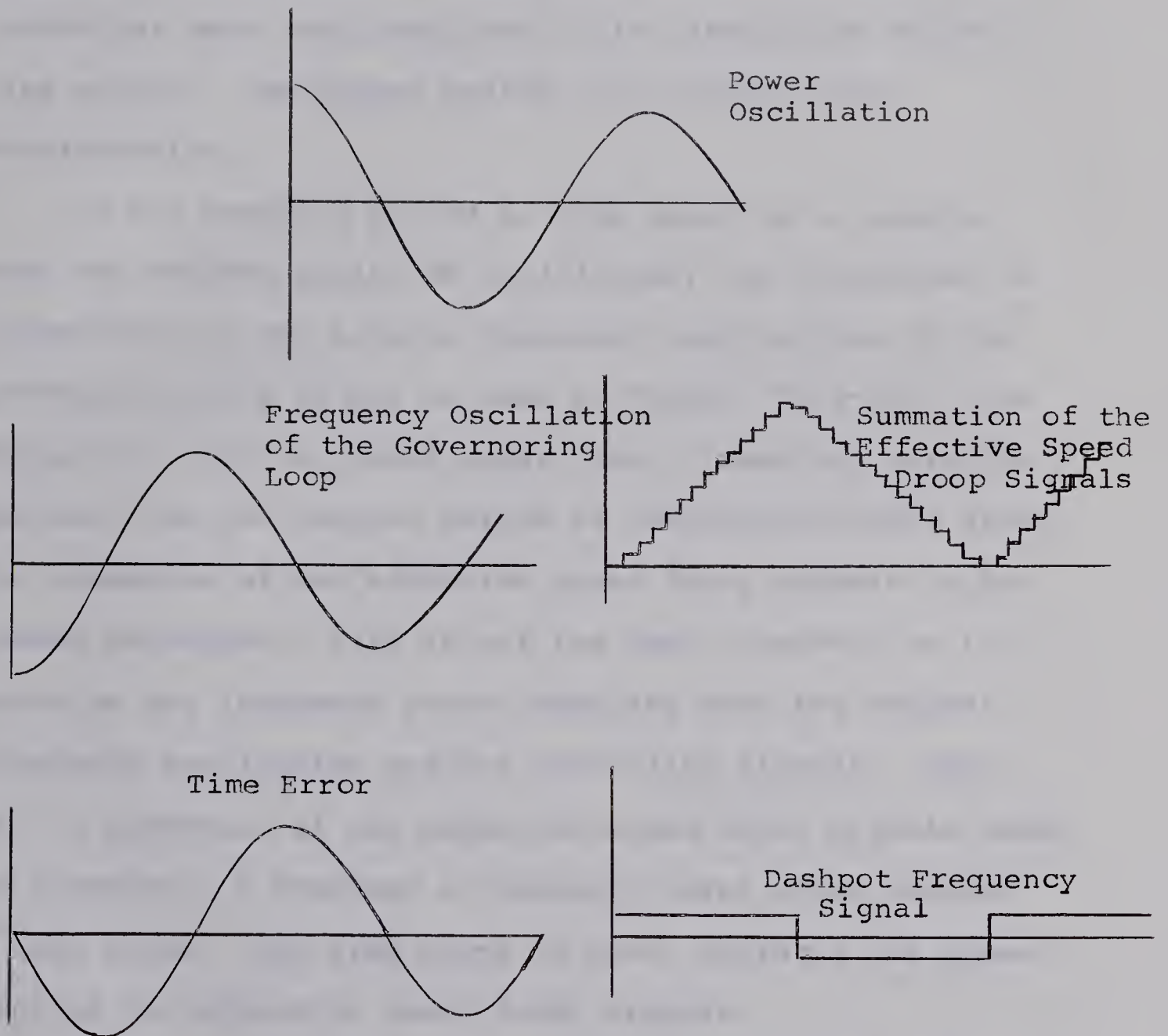


Figure III-1-14

To avoid the above mentioned phenomena, it is necessary to make the controllers insensitive to the natural frequency oscillations. This may be accomplished in either of two ways, by limiting the sampling rate or by increasing the controller dead zone such that it is insensitive to the time errors. The former method will receive first consideration.

If the sampling period is made equal to or greater than the natural period of oscillation, the controller is insensitive to the natural frequency oscillations of the governing loop as may be seen by Figure III-1-15. From Figure III-1-15, it would appear that a sampling duration greater than the natural period of oscillation would allow the summation of the effective speed droop signals to increase unchecked. This is not the case, however, as in practice the frequency error comprises both the natural frequency oscillation and the controller signals. Thus if the summation of the effective speed droop signals tends to increase, it produces a frequency error which creates a time error. The time error in turn, corrects the summation of the effective speed droop signals.

By increasing the dead zone, the controller can also be made insensitive to these oscillations. The amount by which it would be necessary to increase the dead zone (seconds), assuming a sinusoidal oscillations of period 10 seconds, is given by equation III-1-32.

(Peak magnitude of the frequency
oscillation in c.p.s.) (0.637) (5/60) (III-1-32)

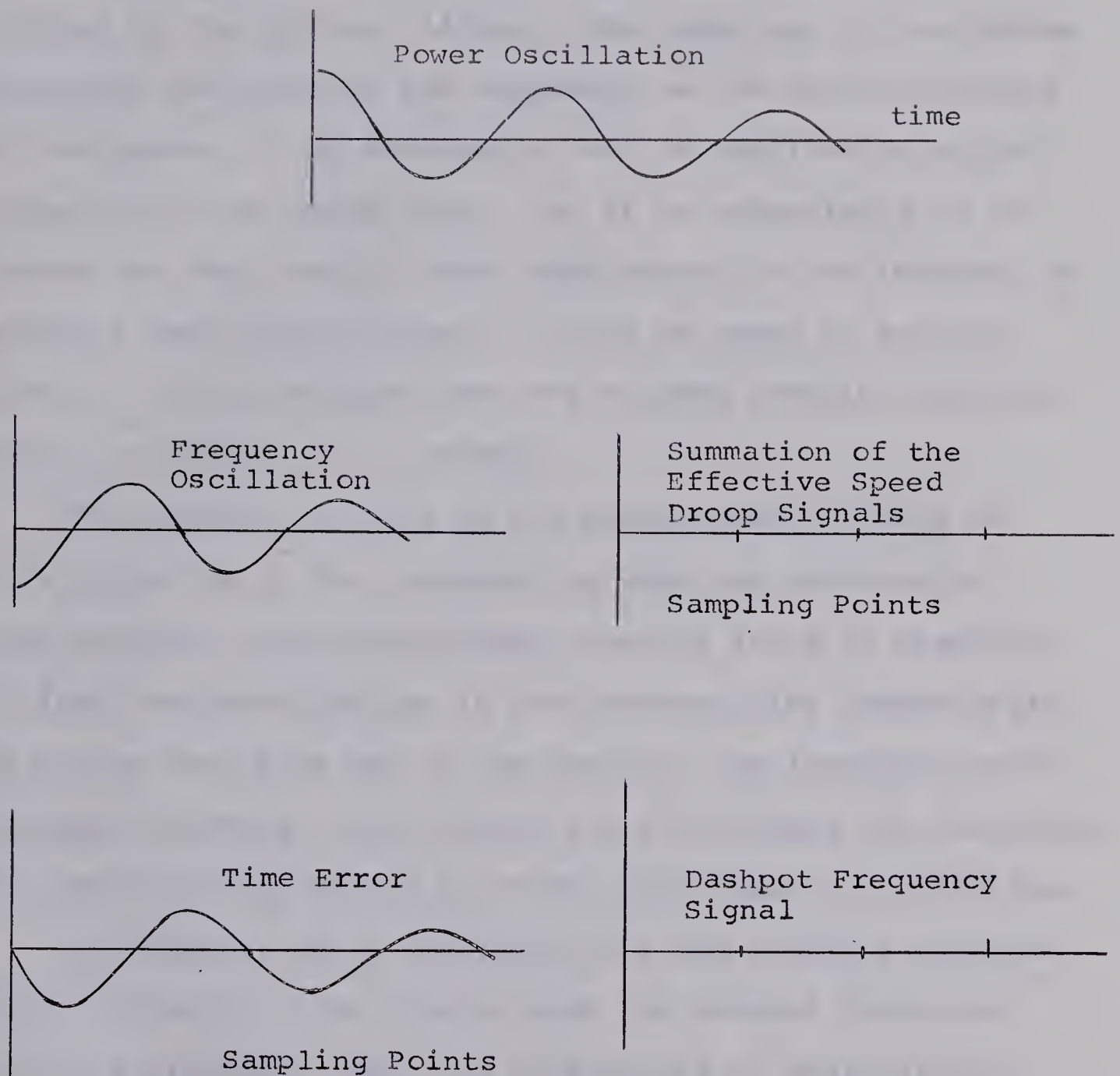


Figure III-1-15

As may be calculated from equation III-1-32, it would be necessary to increase the dead zone by 0.055 and 0.0275 seconds if a 1 and a 1/2 c.p.s., oscillation, respectively, occurred on the system. Although the magnitude of the system frequency oscillations are dependent on the characteristics of the system, it is realizable that an oscillation of the latter magnitude would occur. As it is undesirable to increase the dead zone by these magnitudes, in the interest of accurate load apportionment, as will be shown in Section III-1-5, it is concluded that the minimum sampling duration should be limited to 10 seconds.

The computer results do not exhibit oscillations of this nature as it was necessary to damp the governing loop response, above the values normally found in practice, to limit the oscillations in the synchronizing torque angle. As may be seen from any of the results, the frequency oscillations following a load change are over-damped and therefore, the controller is unable to create this type of oscillation.

In summary, it is concluded that the sampling duration should be equal to or greater than the natural frequency of the governing loop. As this period of oscillation, for a waterwheel generator, is known to be approximately 10 seconds, a sampling duration of 10 seconds is optimum. (20)

Section III-1-5 The Effect of Dead Zone and Backlash

It was shown in Section III-1-3 that it is necessary to incorporate a dead zone equal to or greater than $\pi/2$, to eliminate oscillations in the time and frequency errors. Utilizing the computer results, it will now be shown that

backlash, which was assumed inherent in the controller, has the potential of creating sustained time and frequency error oscillations. These oscillations may be removed by increasing the dead zone such that it is equal to or greater than $km/2$ plus the backlash.

After it is established that oscillations are created by backlash, a detailed investigation of the controller will be made to justify the physical existence of backlash and to indicate where dead zone may be employed to eliminate its affect. Following this the detrimental effects of the required dead zone, on the time keeping capability and load sharing feature of the controller, will be determined.

When the controller model was simulated, a backlash signal of 0.02 seconds was assumed, which affected the operation of the controller, in the absence of dead zone, as follows. With a negative time error the backlash increases the signal activating the controller by 0.02 seconds. When the controller corrects the latter signal, the sign of the former is reversed making the activating signal 0.02 seconds positive.

As backlash introduces a disturbance, it may be considered as an energy source in the controller. In the absence of dead zone this phenomena produces sustained oscillations in the time and frequency errors, as shown in Figures III-2-11 to III-2-13 of the computer results.

The effective speed droop signal is unable to correct a time error either existing or introduced in the system. Rather, its purpose is to correct a system frequency error.

As the backlash is a time error rather than a frequency error, it must be counteracted by the dashpot frequency signal. Further, the magnitude of the oscillations in the time and frequency errors will decrease from a large value, or increase from a small value, until the time error dissipated by the dashpot frequency signal is equal to that introduced by backlash.

The dashpot frequency signal, existing in the controller, was 0.042 and 0.082 c.p.s. for the load changes shown in Figure III-2-11 and Figures III-2-12 and III-2-13 respectively. As this signal is applied continuously during any half-cycle period of the oscillation, the time in seconds required to dissipate the backlash is given by equation III-1-33.

$$t = \frac{(B_L) (60)}{V} \quad \text{(III-1-33)}$$

Thus for the backlash simulated in the computer model (0.02 seconds), approximately 30 and 15 seconds are required to dissipate the energy introduced by the respective dashpot frequency signals. The former period of oscillation (60 seconds), which is incurred with a 10 second sampling duration, agrees with the average period of oscillation found in the computer results. However, the computer results show that the latter period of oscillation is 40 rather than 30 seconds. This difference is believed to be caused by two factors. The dashpot frequency signal is applied through the governor dashpot, therefore, on account of the dashpot relaxation time constant (T_r), its affect is neither applied to

or removed from the system when the controller applies the dashpot frequency signal to, or removes it from, the governor. Second, as the governing loop response is overdamped, the system frequency does not follow (instantaneously) the summation of the effective speed droop signals. Thus a longer period of time is taken to counteract the backlash signal. Both of these factors have a larger affect with the more rapid sampling rates; in the former case the time taken to apply or remove the dashpot frequency signal is greater with respect to the sampling duration, in the latter case the response rate of the controller increases with the more rapid sampling rate.

If the dead zone is made equal to or greater than $km/2$ plus the backlash, the time and frequency error oscillations are eliminated. This may be seen by observation of Figures III-2-1 to III-2-10 of the computer results, where the dead zone is equal to 0.05 seconds.

To determine where backlash may originate in the controller and how its affect may be eliminated by dead zone, the controller is sectionalized as shown in Figure III-1-16.

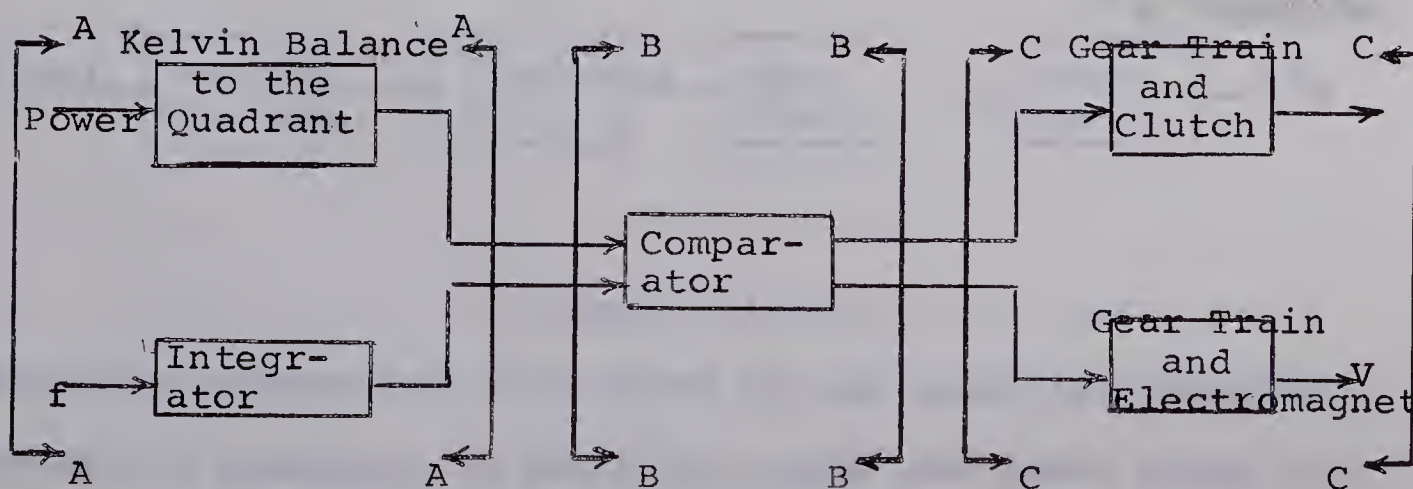


Figure III-1-16

Section A-A incorporates the integrator and the load sharing feature of the controller. The integrator is a 6 R.P.M. telechron synchronous motor while the load sharing feature is composed of; the Kelvin Balance, a reversing motor, and the gear train which drives the quadrant.

A telechron motor runs at 3,600 R.P.M., however, its output speed is reduced through a gear train. Any backlash between the system frequency and the rotating arm of the controller must exist in the gear train of the motor. A synchronous motor may not be separated from the system frequency by more than 90 electrical degrees, which is equivalent to a time error of approximately 4×10^{-3} seconds at the rotating arm of the controller. Assuming a tolerance of 1/1000 in the gear train of the motor, a backlash of 0.01 seconds could be incorporated in the controller at this point, as it takes 10 seconds for the arm to make one revolution.

The block schematic of the load sharing feature is shown in Figure III-1-17.

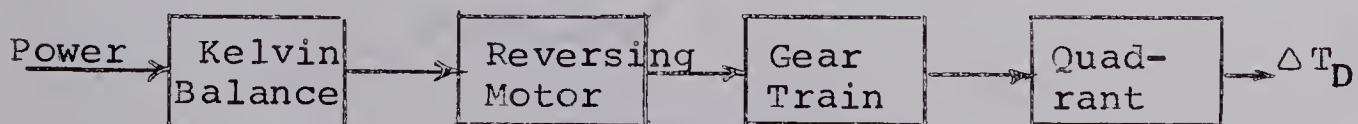


Figure III-1-17

Any play between the gear drive of the load bias reversing motor and quadrant, on which the raise and lower relay contacts are mounted, would allow backlash in this section

of the apparatus. Assuming $2/1000$ tolerance in the gear train, the amount of backlash incorporated would be $2/1000$ of the maximum time droop bias signal ($5/8$ seconds). This is equivalent to a time backlash of 0.00125 seconds.

Section B-B of the controller apparatus, shown in Figure III-1-18, will now be considered. This incorporates the controller action from the load bias and electrical time signals, to the command signals given to Section C-C of the controller.

This section of the apparatus will not contain backlash, as it is composed of relays which are solidly attached to the quadrant which have little resilience or inertia. It is possible, however, to incorporate the desired insensitivity or dead zone in this section of the apparatus.

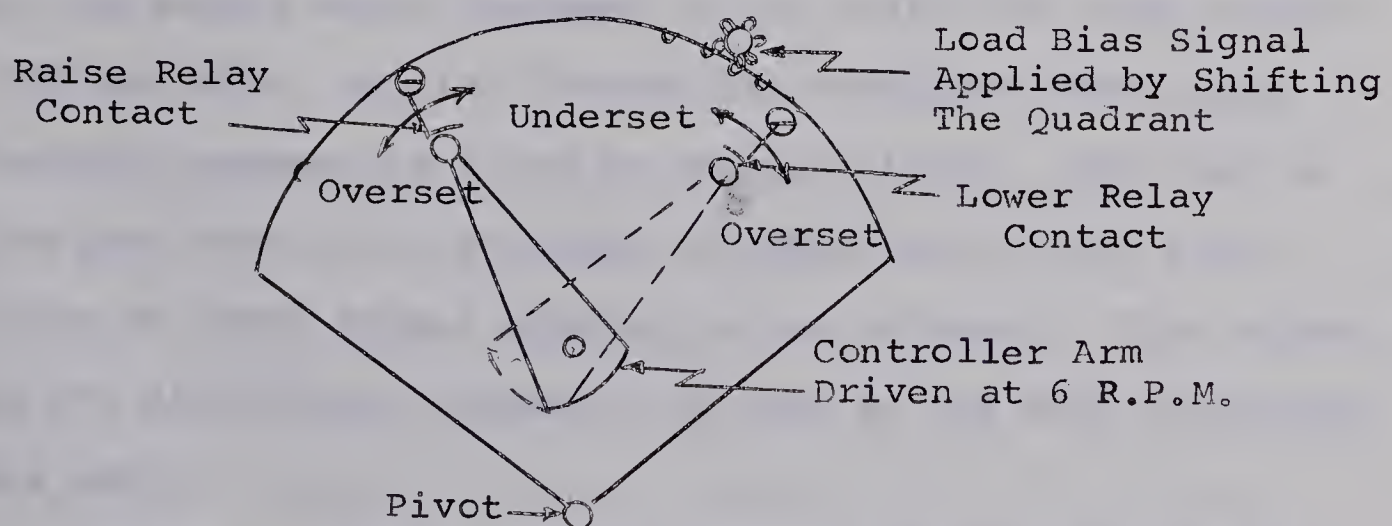


Figure III-1-18

Assuming that the electrical time and load bias signals are correct, the master clock signal should initiate and terminate immediately after, and immediately before, the controller opens and closes the raise and lower relay contacts. If the contacts are under set, false time errors

will be generated and if they are over set, dead zone will be incorporated in the controller.

Section C-C, which incorporates the controller operation from the comparator to the point at which the signals are applied to the governor, will now be considered.

The method by which the effective speed droop signal is applied to the governor is shown in Figure III-1-19.

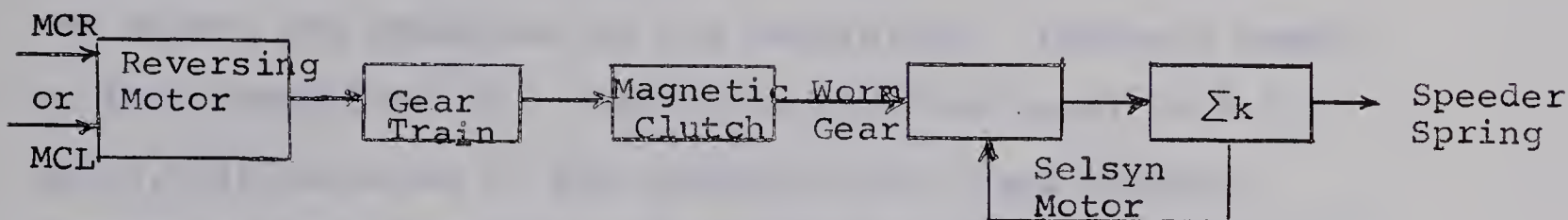


Figure III-1-19

The backlash in the above apparatus is insignificant as the selsyn motor responds to the raise and lower signals (MCR and MCL), applied through the reversing motor, and thereby removes the force on the gear train. The play in the gear train will decrease the magnitude of the first raise or lower signal applied to the governor. This affect is not significant, however, as long as the gear tolerances are small.

The method by which the dashpot frequency signal is applied to the governor is shown in Figure III-1-20.

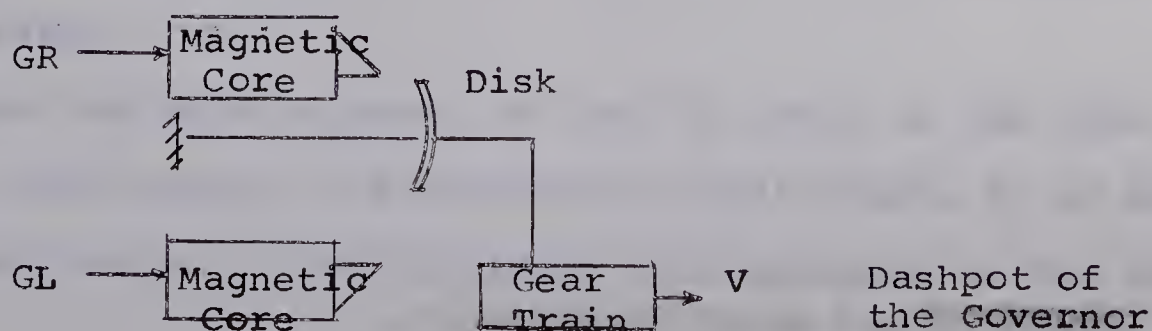


Figure III-1-20

The backlash in this apparatus is also thought to be insignificant due to the damping in the dashpot of the governor. When a raise or lower signal (GR or GL) is applied to or removed from the governor, the inherent damping of the dashpot prohibits any rapid variation in the magnitude of the signal. Thus backlash may not exist.

From the former analysis, it is concluded that only the backlash in the integrator and load sharing feature can affect the operation of the controller. Further, based on the assumptions of a time error backlash equal to 0.01 and 0.00125 seconds in the integrator and load sharing feature of the apparatus, the required dead zone for optimum controller operation is given by equation III-1-34.

$$\begin{aligned} \text{Required Dead Zone} &= 0.01 + 0.00125 + \frac{km}{(2)(60)} \\ &= 0.01 + 0.00125 + \frac{\delta p}{600} \end{aligned} \quad (\text{III-1-34})$$

Assuming a 2 and 4 per cent permanent speed droop setting, the required dead zone is calculated to be 0.01325 and 0.01525 seconds respectively. For ease of calculations, the required dead zone is approximated as ± 0.015 seconds. This required dead zone (insensitivity of the controller) incorporates an error in the load division between generating units and, of course, allows a time error of ± 0.015 seconds on the system.

As the apportionment of load is based on the time droop bias signal (5/8 seconds for full-load), it is expected that the error in load division will increase as the individual

generator load decreases. The maximum possible value of this error is given by equation III-1-35 and is shown in Figure III-1-21.

Possible Error in Machine Load =

$$\frac{M_{nr}}{M_n} \frac{0.030}{5/8} = \frac{M_{nr}}{M_n} 4.8\% \quad (\text{III-1-35})$$

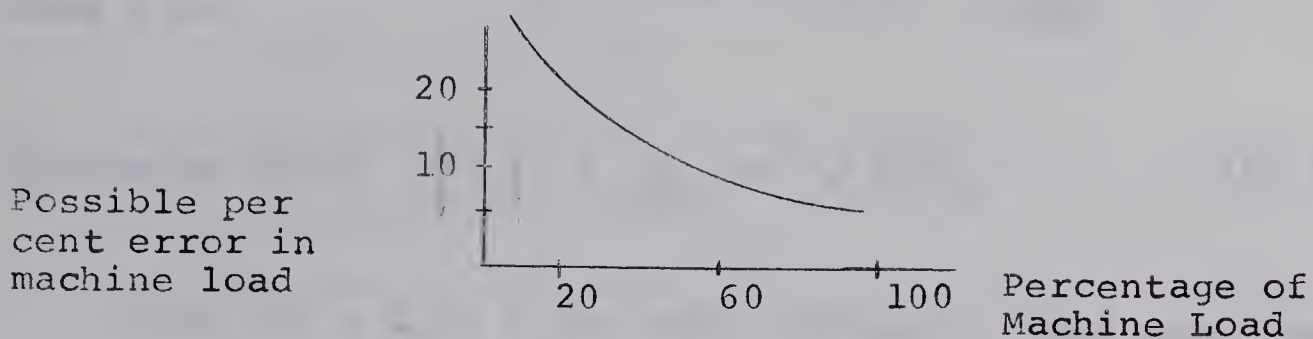


Figure III-1-21

As it was found necessary to increase the absolute value of the dead zone to 0.030 seconds, to make the controller insensitive to backlash, the controller will not respond to a frequency error created by a load change until the activating time error exceeds the fore-mentioned value. At this time a frequency error will exist on the system which will act to increase the time error. Considering a ramp load change within the maximum response of the controller, the largest frequency error will be accumulated for the most rapid load change. This is the case as the time error is the integral of the frequency error.

The frequency error which the controller must correct, existing when the activating time error is equal to the dead zone, will not significantly degrade the operation of the controller if it is not larger than 3 or 4 per cent of

the permanent speed droop setting. For a ramp load change equal to one-half the maximum response of the controller, assuming that a positive time error of 0.015 seconds existed when the load change began, the frequency error on the system when the time error is equal to the dead zone is given by equation III-1-36.

$$\begin{array}{l} \text{Absolute Value of the} \\ \text{Dead Zone} \end{array} = 0.030 = 1/2 \delta_p \frac{1}{1000} t^2$$

$$\text{Frequency Error } (\%) = \frac{t}{10} = \sqrt{36/\delta_p} \quad (\text{III-1-36})$$

Thus for a 2 or 4 per cent permanent speed droop setting, a step frequency error of 5.5 and 3.9 per cent respectively is applied to the controller when the time error is equal to the dead zone (-0.015 seconds). It is to be realized that these values represent the worst case as, if a zero initial time error were assumed, they would reduce to 3.9 and 2.7 per cent respectively.

With a 2 per cent permanent speed droop setting, it may be realized that it is desirable to decrease the backlash below ± 0.015 seconds. However, as may be seen from Figure III-1-21, it is desirable to minimize the dead zone regardless of the permanent speed droop setting in the interest of accurate load division between the generating units. This latter factor is thought to be the dictating requirement, as otherwise the controller's 20 per cent load setting would have little meaning.

In summary, it is desirable to minimize the dead zone and accordingly the backlash to provide accurate load division among the generating units and yet, prevent sustained oscillations on the system.

It has now been determined that the controller performance may be optimized for load changes equal to or less rapid than one-half the maximum response of the controller.

Section III-1-6 Conclusion

The sustained oscillations in the frequency and synchronizing torque angle were found to have a justified existence if the generator inertia constants are unequal. Although this was not the intention in the computer simulation, it is expected that slight inaccuracies in the computer potentiometer settings resulted in unequal inertia constants which produced the oscillations.

The controllers could not be optimized for load variations more rapid than one-half their maximum response. As it is necessary to incorporate a time dead zone, to eliminate sustained time and frequency oscillations, a frequency error is accumulated before the controllers are activated by a time error. To correct this frequency error and thereby prevent an increasing time error, the load change must not exceed the former limit. The maximum response rate of the controllers, which is their maximum ability to correct the frequency error created by the load change, was found to be dependent on the magnitude of the effective speed droop signals and the sampling rate.

With all generators on 100 per cent load control, it was found that the magnitude of the controller's signals, as related to the respective speed droop settings of the individual generators, should be constant. However, it was found unnecessary, although possibly preferable, to reduce the magnitude of the controller signals, on a given unit, as the percentage load setting is reduced on that unit. This was found to be the case as a lower percentage load setting reduces the required controller response.

The optimum magnitude of the controller's signals were found to be dependent on the characteristics of the power system since the controllers apply a disturbance to the system. The power output of the generators varies in accordance with the frequency error seen by the governors. In turn, the frequency error seen by the governor varies in accordance with the position of its respective permanent speed droop curve. As the effective speed droop signal varies the position of the permanent speed droop curve and the dashpot frequency signal applies a frequency error to the governor, the controller applies a power disturbance to the system, via the governor, when it responds to a time error. To limit the magnitude of this disturbance, it is the opinion of the author that the magnitude of the effective speed droop signal should be limited to 2 per cent of the permanent speed droop setting. As it was found that the dashpot frequency signal should be equal to one-half the effective speed droop signal, the power disturbances will not exceed 3 per cent of the station or

generator capacity.

It was found that the sampling period should be equal to or greater than the natural frequency of oscillation of the governing loop, otherwise the controllers are able to create sustained frequency oscillations on the system. This period of oscillation is known to be approximately 10 seconds from the literature published (20). Therefore, in the interest of maximizing the controller response, a 10 second sampling duration is optimum.

As the controller is made up of mechanical elements, it was assumed that backlash was inherent in its operation. It was found that this backlash has the potential of creating sustained frequency and time error oscillations. To eliminate its affect, it is necessary to increase the dead zone to encompass the backlash. The dead zone, however, was found to have a detrimental affect on the load sharing feature, as the latter apportions the load on a time droop basis. The possible error in the load carried by the individual unit, assuming a dead zone of ± 0.015 seconds, was found to be 4.8 per cent at full-load and to vary inversely with load.

Section III-2 The Computer Results

In presenting the computer results, the following symbols are defined:

Δf	the system frequency expressed in c.p.s.
ΔT	the time error on the system expressed in seconds
$\Delta T'_1$	the activating time error of unit #1 expressed in seconds
$\sum k_1$	the summation of the effective speed droop signals of unit #1, expressed as percentage of its respective permanent speed droop curve
$\sum k_2$	the summation of the effective speed droop signals of unit #2, expressed as a percentage of its respective permanent speed droop curve
P_{T1}	the turbine output power of unit #1 expressed in per unit
P_{T2}	the turbine output power of unit #2 expressed in per unit
δ	the synchronizing torque angle between machines expressed in degrees (assumed positive when unit #1 is leading unit #2)

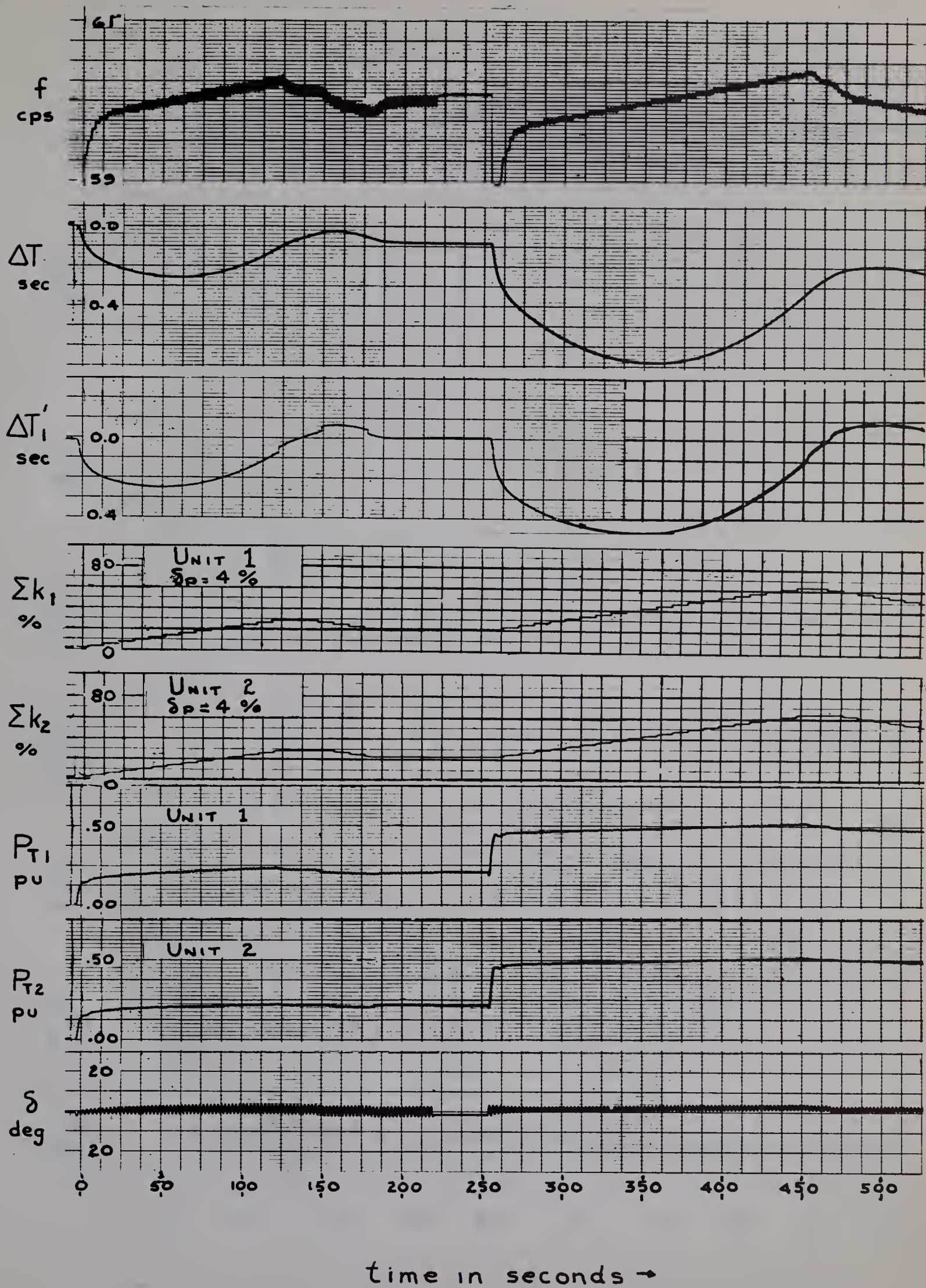


FIGURE III-2-1 (TEST 2)

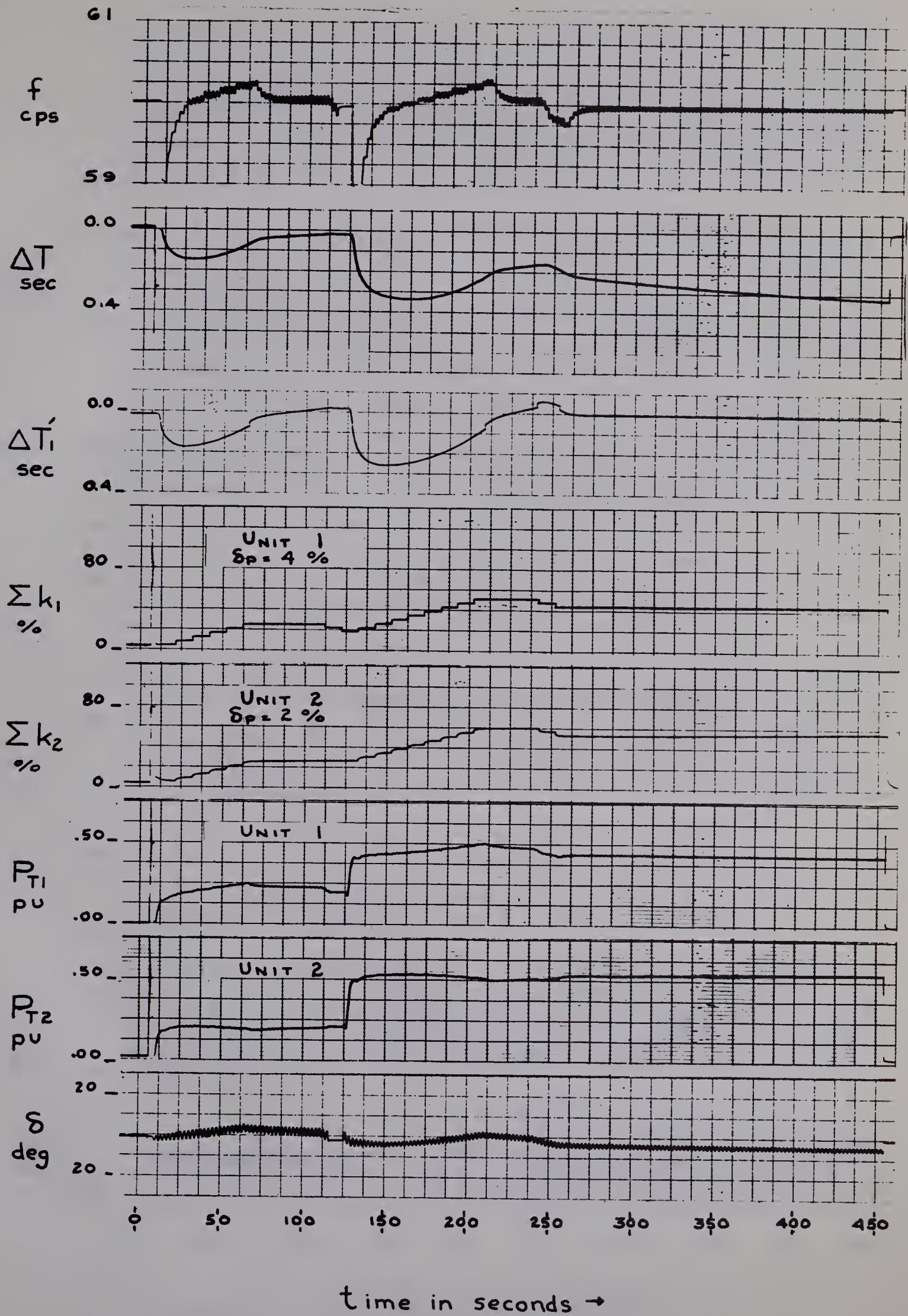


FIGURE III-2-2 (TEST 5)

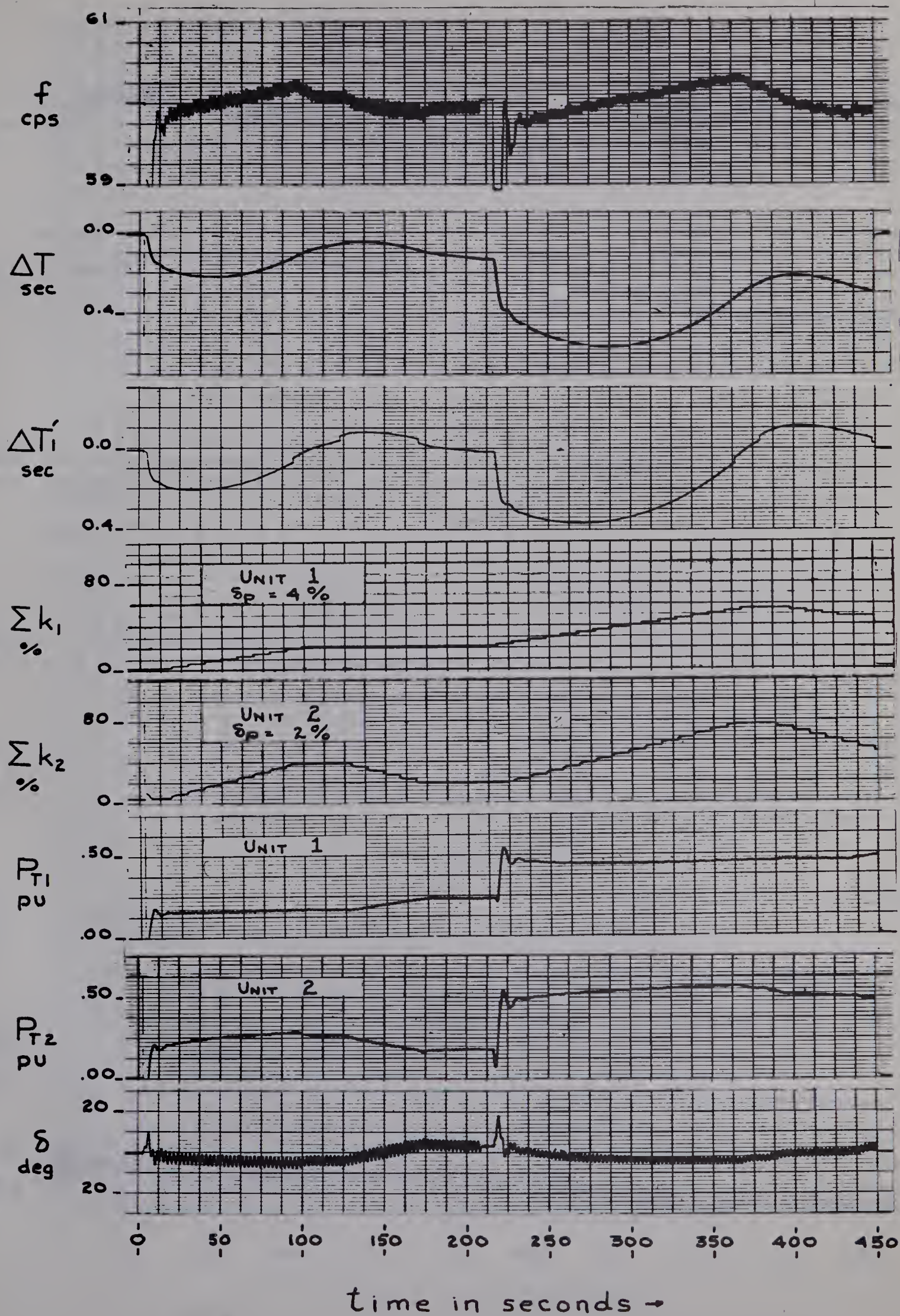


FIGURE III-2-3 (TEST 8)

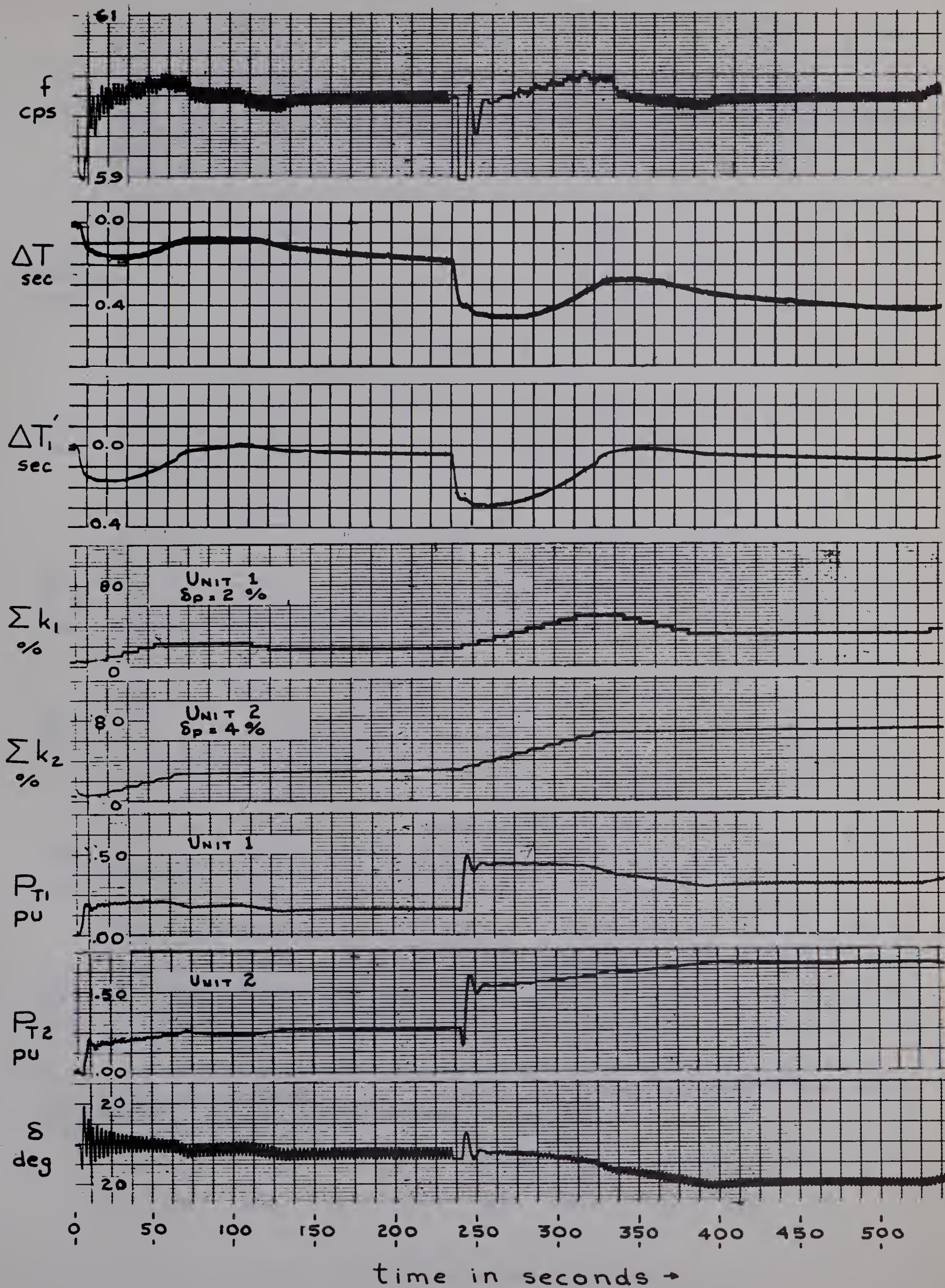


FIGURE III-2-4 (TEST 13)

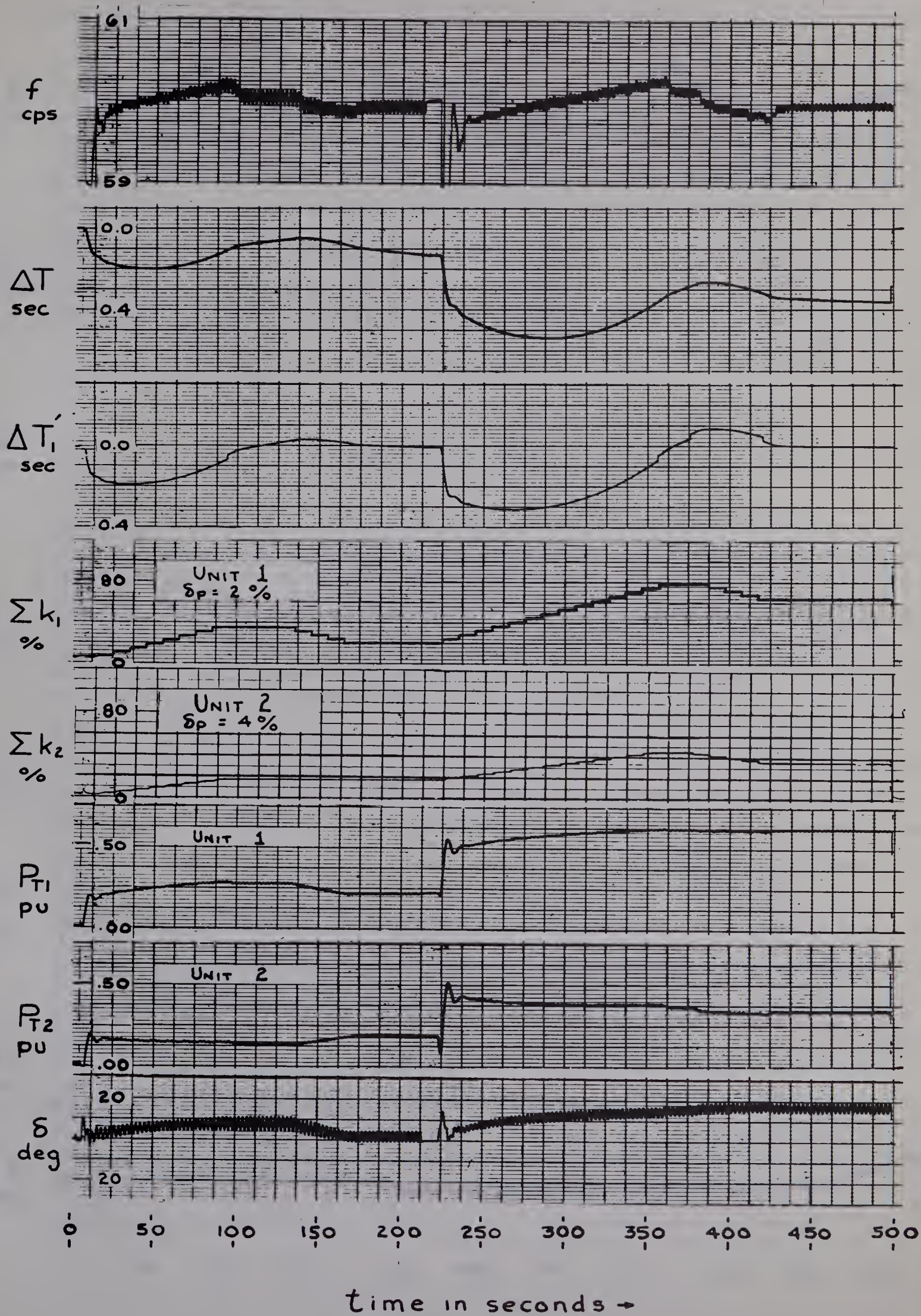


FIGURE III-2-5 (TEST 12)

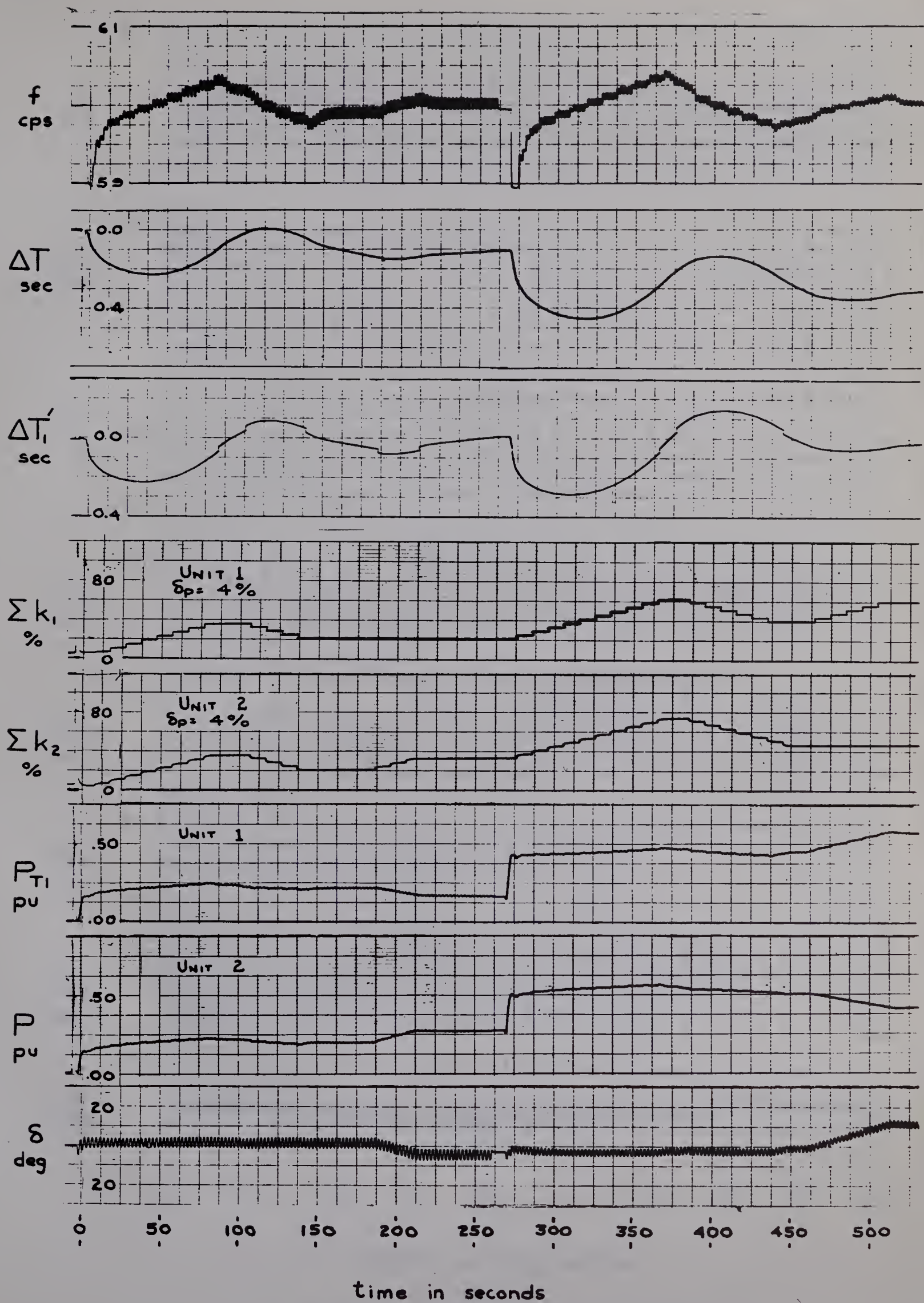


FIGURE III-2-6 (TEST 15)

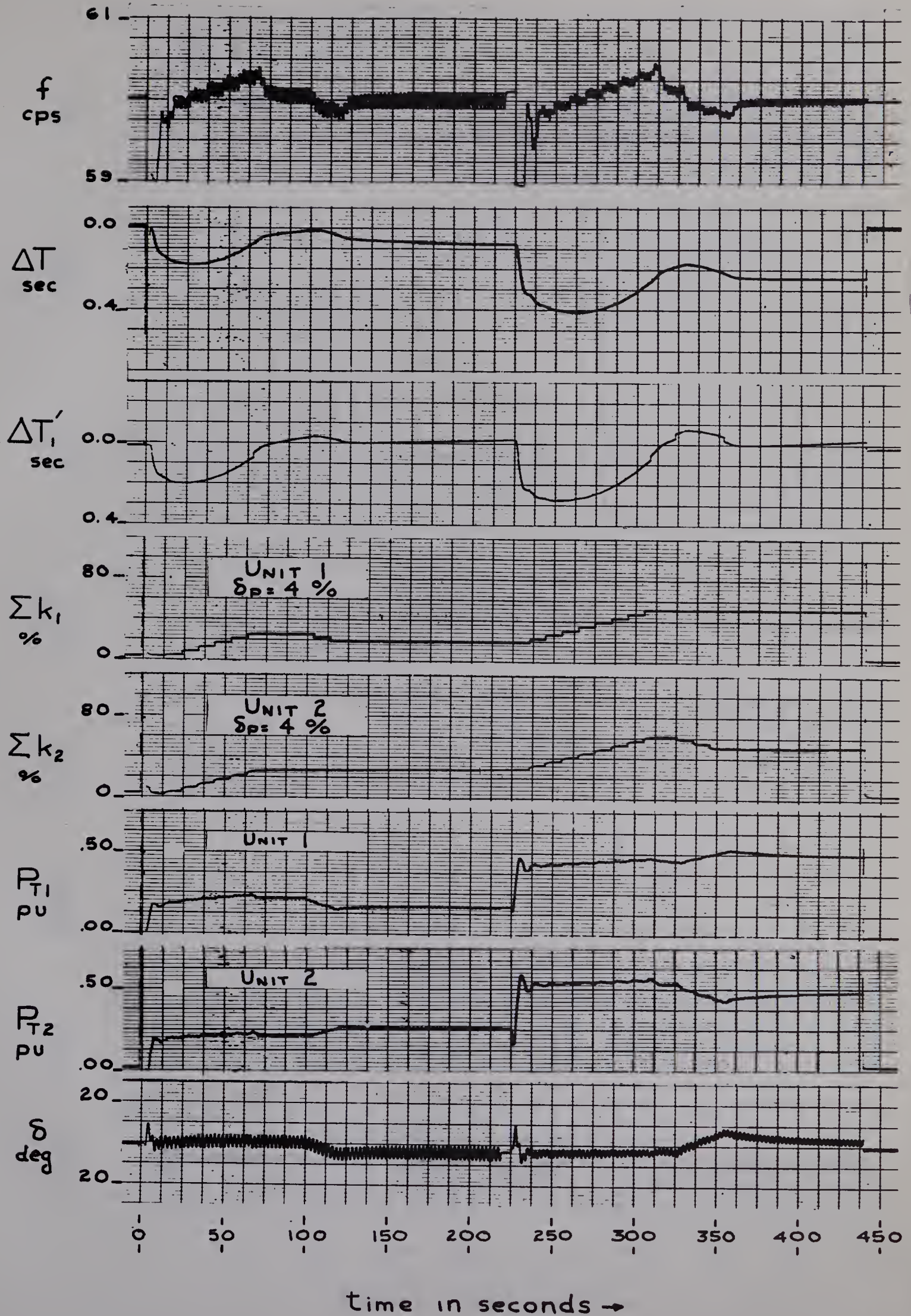


FIGURE III-2-7 (TEST 16)

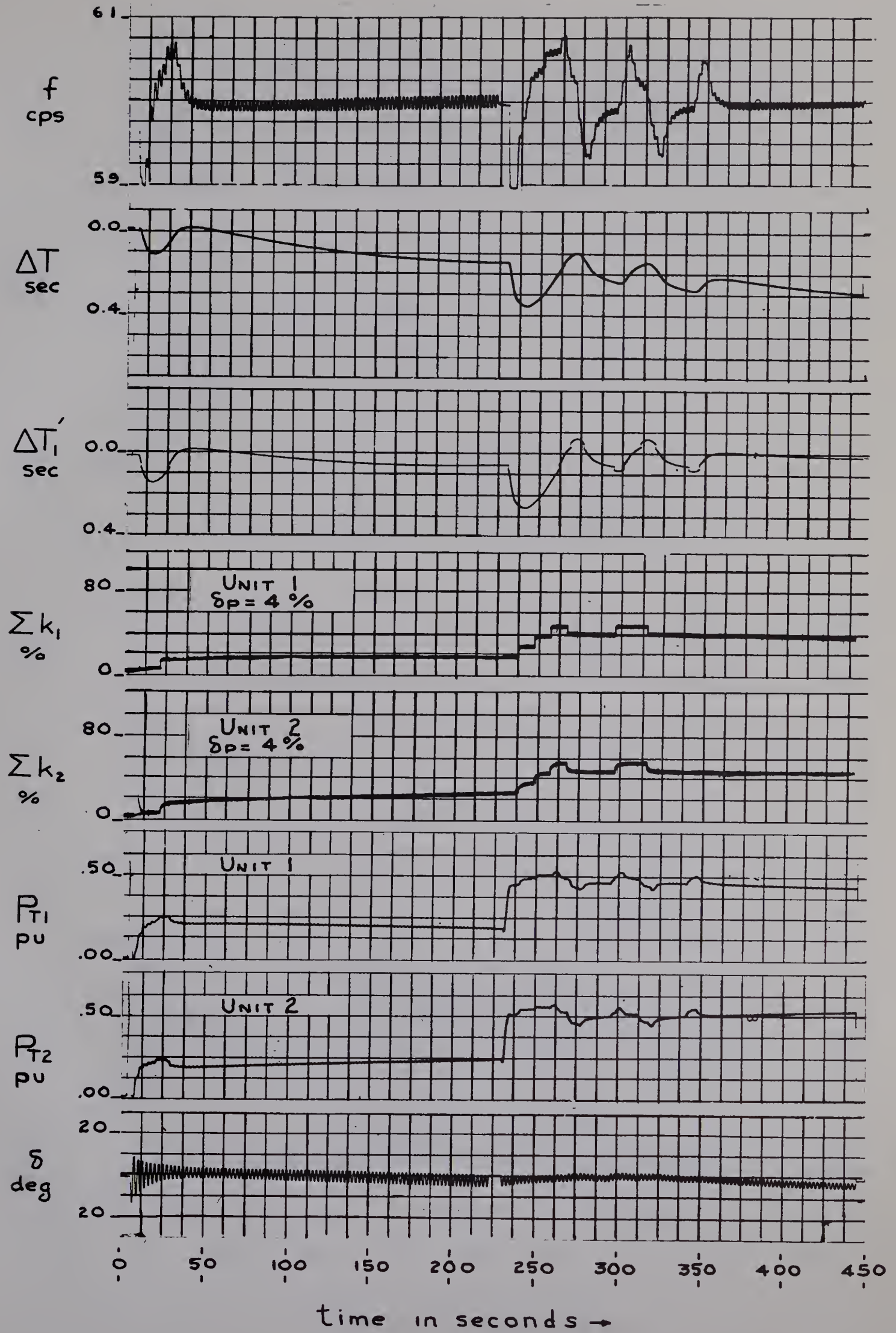


FIGURE III-2-8 (TEST 21)

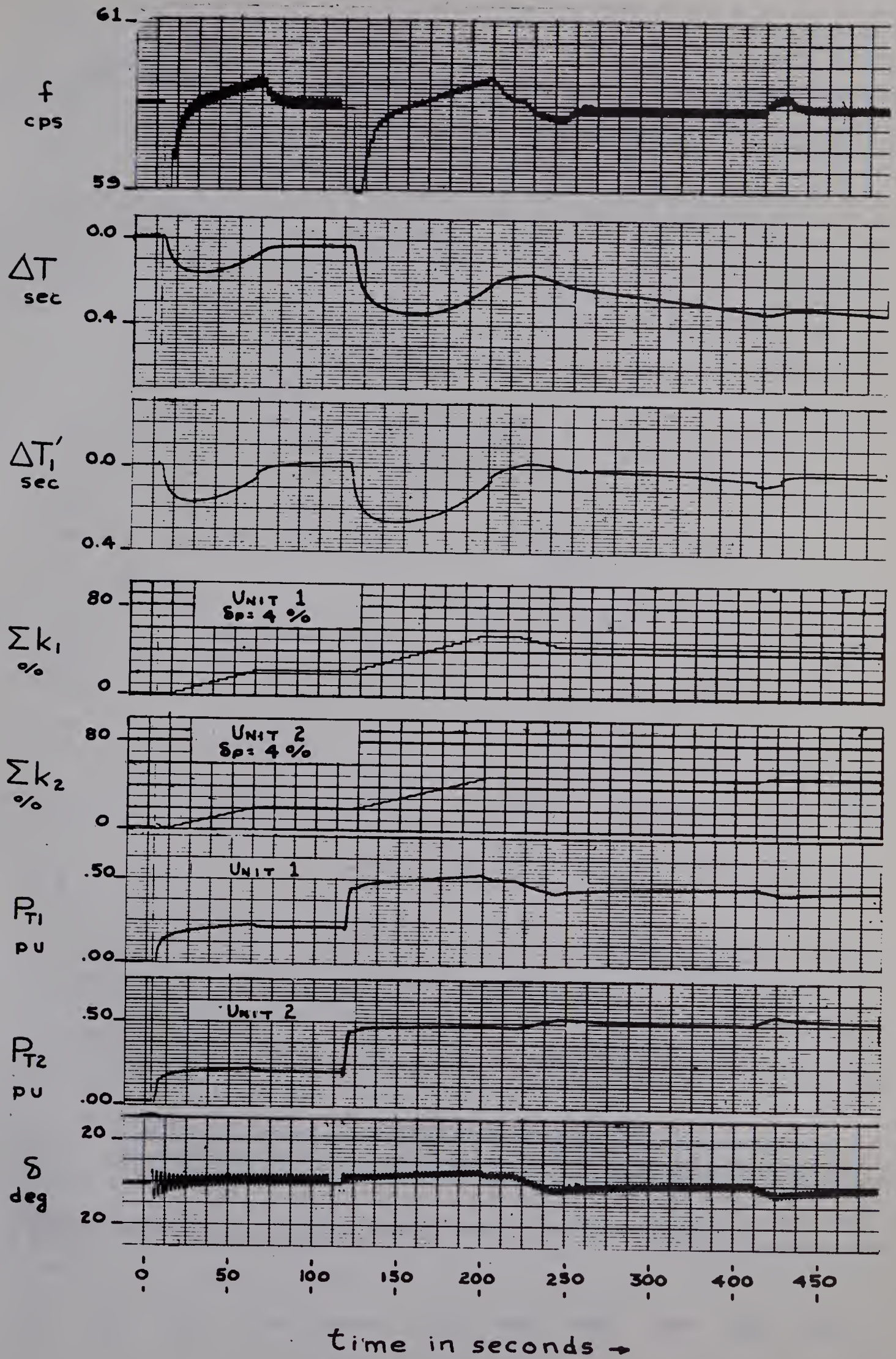


FIGURE III-2-9 (TEST 22)

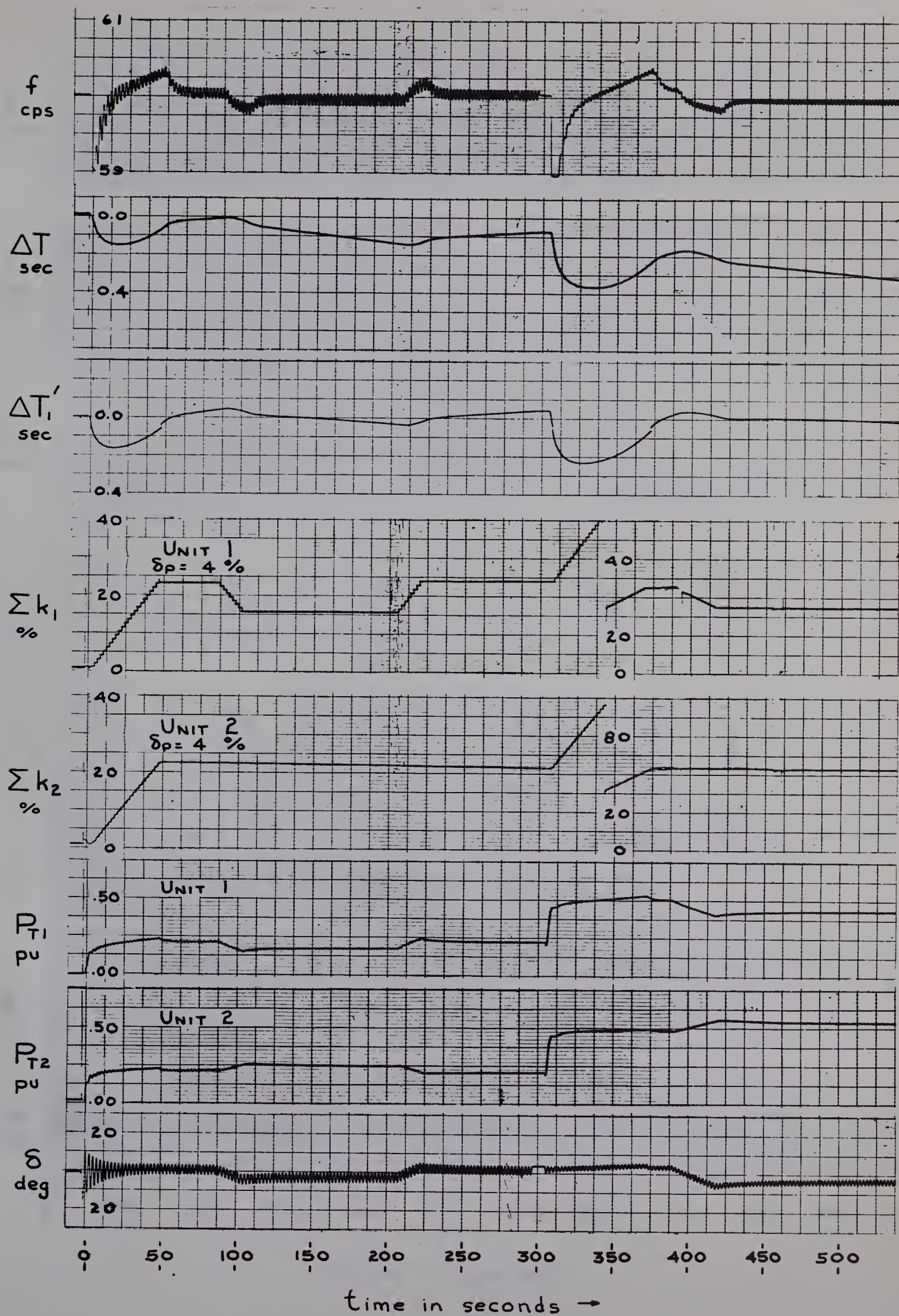


FIGURE III-2 10 (TEST 25)

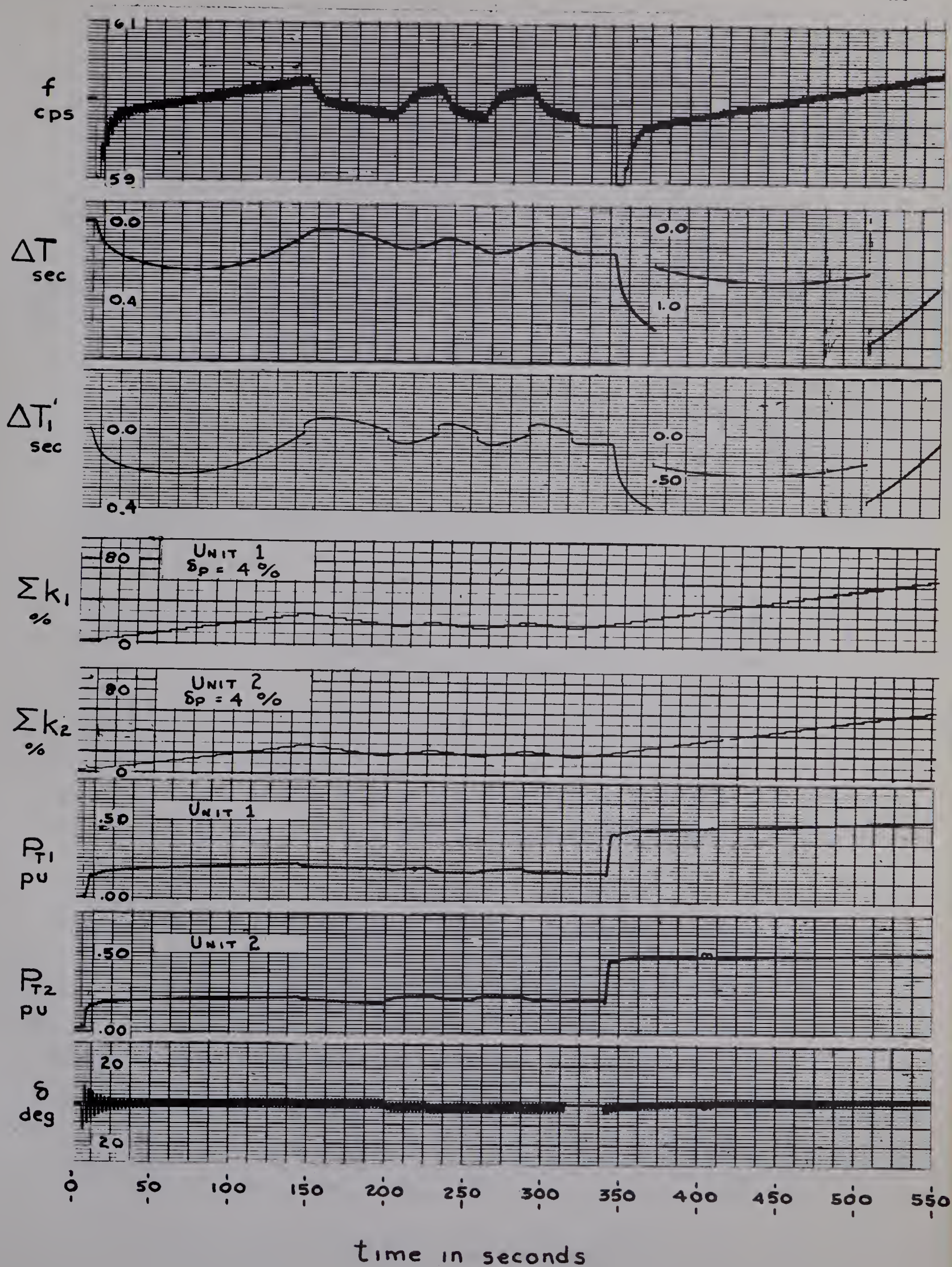


FIGURE III-2-11 (TEST 28)

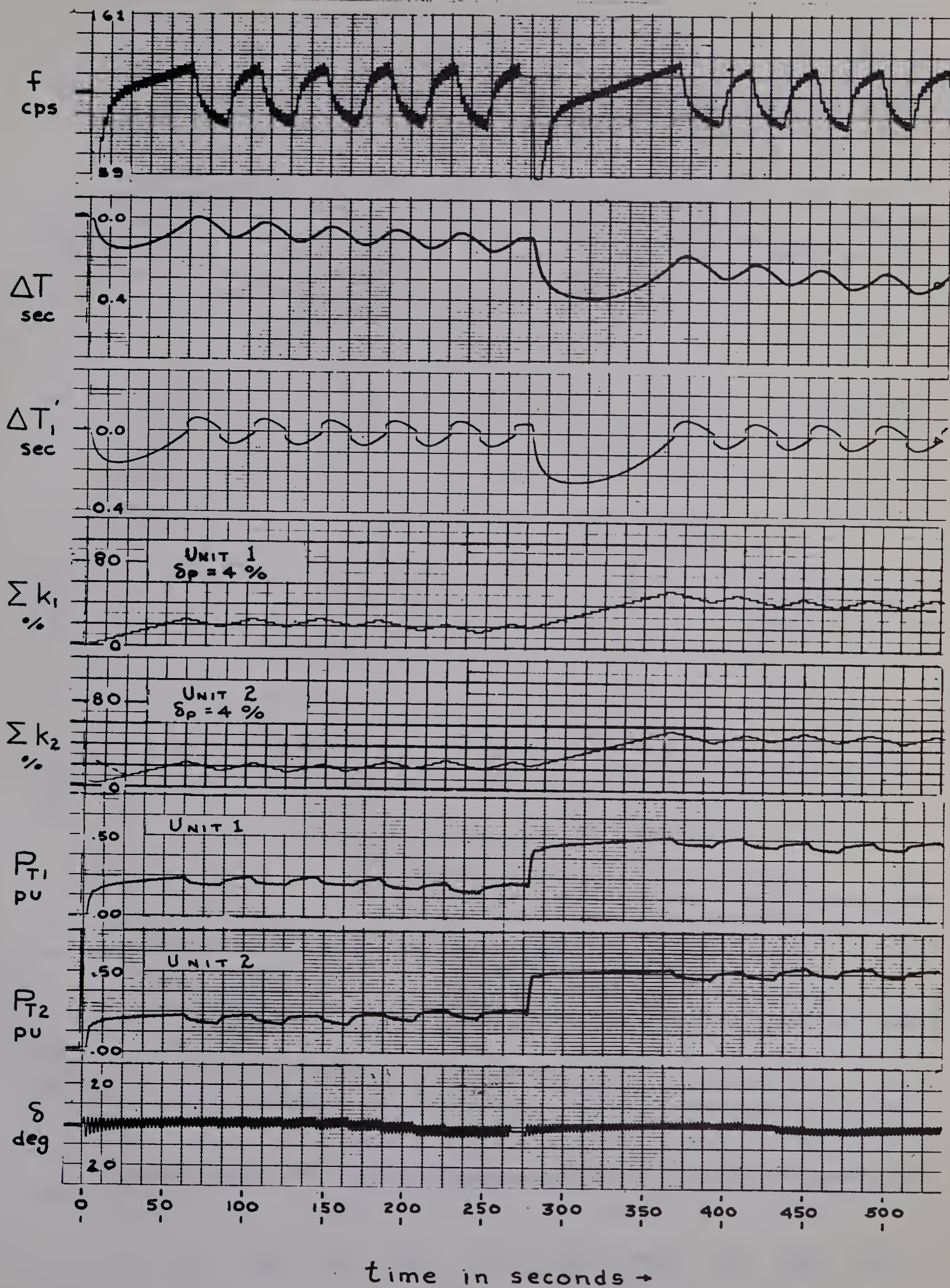


FIGURE III-2-12 (TEST 29)

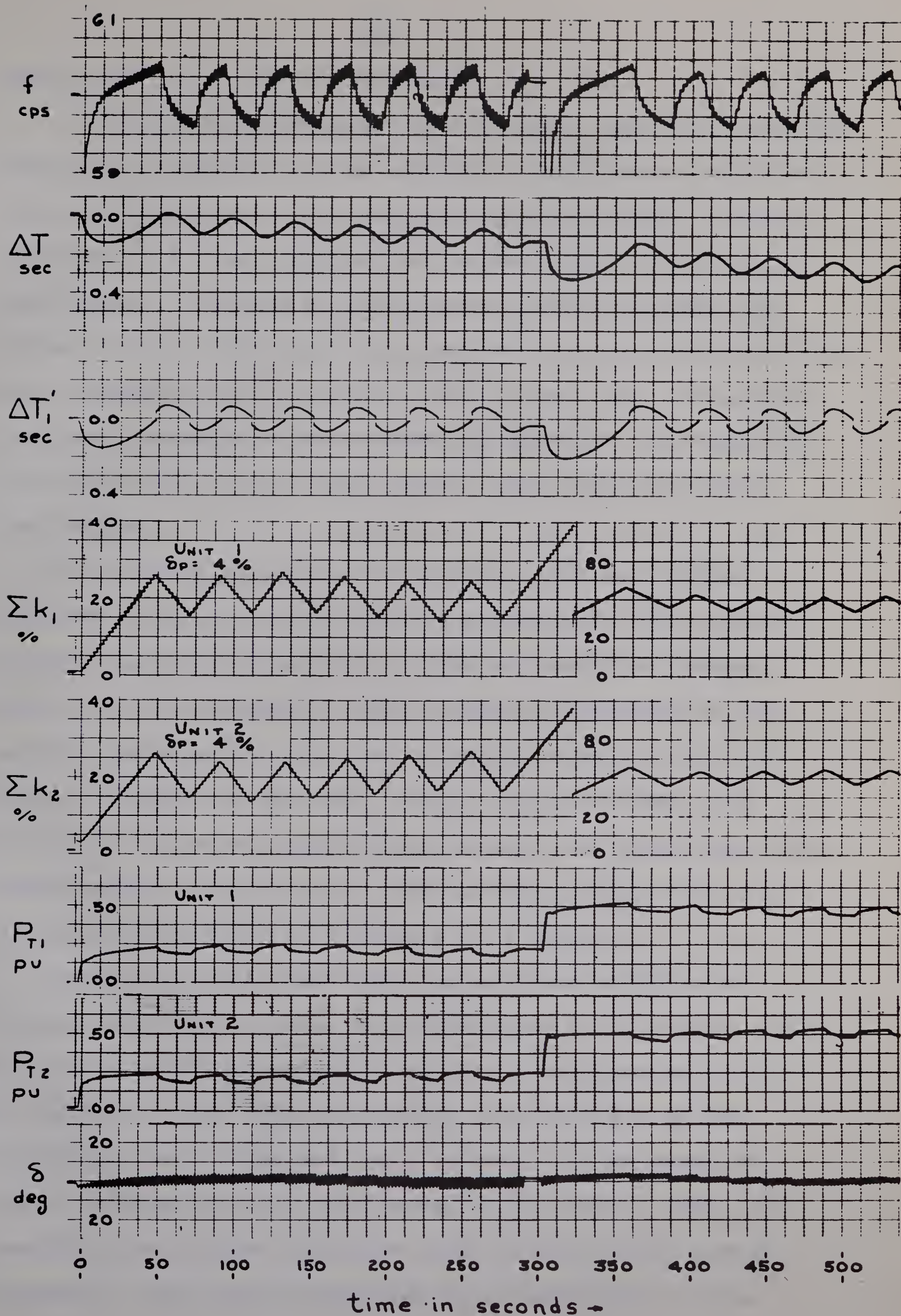


FIGURE III-2-13 (TEST 30)

Section III-1-3 General Conclusion

The generator, governor, hydro-system, load and controller models were simulated on the computer as developed in Sections 1 to 5 of Chapter II. The voltage regulator model, developed in Reference 4, was accepted and incorporated with the generator model. The R.M.S. magnitude of system voltage, the variation from system frequency, as well as the other quantities were represented by proportional D.C. quantities. The system simulated consisted of two generating stations, approximately 150 miles apart, with a load concentrated half-way between the stations.

The voltage variations in the generator and voltage regulator models were not found to be adversely affected by the operation of the controller, for the sampling periods tested (2 to 10 seconds). This result was expected as the natural frequency of this loop is approximately 0.5 to 1.0 c.p.s. Further for the same reason, the controllers did not affect the power oscillations between the generating units (synchronizing torque loop). The natural period of this oscillation was found to be from 1 to 3 seconds.

However, it was found that the sampling periods, of the individual generators, should be equal to or greater than the natural period of oscillation of the governing loop (10 seconds) (20). The governing loop consists of the generator, the turbine and the governor. If the sampling rate is increased above this value, it is possible for the controller to produce sustained oscillations in the system frequency. This may be realized as an oscillation in the

system frequency, caused by a power disturbance, creates a time error. The controllers respond to this time error and attempt to adjust the speed droop curves to correct the frequency error. Thus the frequency error, seen by the governor, increases and, as a result, the oscillation in the turbine output power also increases. This in turn increases the frequency error on the system. Therefore, to maximize the controller's response but prevent sustained frequency oscillations on the system, the sampling period should be equal to 10 seconds.

With a sampling period equal to or greater than the natural period of the governing loop, the controller does not affect the transient response of the power system. However, when the controllers adjust the permanent speed droop curves of a power station, a disturbance is applied to the system. The magnitude of this disturbance is directly proportional to the magnitude of the effective speed droop and dashpot frequency signals as they determine, via the governor, the change in the turbine output power. Therefore, to minimize the system power disturbances, it is desirable to limit the magnitude of the controller signals. However, there are also requirements for large effective speed droop and dashpot frequency signals. The former determines the maximum rate of the controller response, for a 10 second sampling duration, and the latter determines the controller damping. In the interest of the foregoing conflicting criteria, it is the opinion of the author that the effective speed droop and dashpot frequency signals should be equal to 2 and 1 per cent of the permanent speed droop setting respectively.

With all generators on 100 per cent load control it was found that the relative magnitude of the controller signals, expressed as a percentage of respective speed droop settings of the units on which the controllers operate, should be equal. Otherwise, following a system load change, the controllers with the smaller signals must respond more rapidly. Thus the maximum response of the controllers is unnecessarily limited. Further, if a per unit ramp load change does not occur in less than 165 seconds on any generator, it was found unnecessary to reduce the magnitude of the controller signals as the percentage load setting is reduced. This was found to be the case as a lower percentage load setting reduced, rather than increased, the required controller response.

The former restrictions were imposed on the controller by the system characteristics. As the controller is also a mechanical device, it has inherent characteristics, which could be detrimental to the power system performance. The controller signals are of constant magnitude and therefore the system time error can only be corrected within the tolerances of the signals, thus it is necessary to make the controller insensitive to time errors below this magnitude or accept small but sustained time and frequency oscillations. Further, assuming a per unit tolerance of $1/1,000$ and $2/1,000$ in the gear drives of the comparator and load sharing feature respectively, a backlash of approximately 0.01 seconds results from the inertia of the components. This backlash has the potential of increasing the magnitude of the former oscillations. To prevent these sustained oscillations from occurring, it

is necessary to make the controller insensitive to time errors less than or equal to approximately ± 0.015 seconds. As these oscillations are undesirable, it is assumed that this dead zone is implemented.

The dead zone has two main detrimental affects. Since the load is divided among the units on a time droop basis, the error in load division varies inversely proportional to load, and is 4.8 per cent at full load. Second, a frequency error must exist on the system, which is proportional to the dead zone and rate of load change, before a time error is created to which the controller can respond. To correct this frequency error in minimum time, the rate of load change must not exceed one-half the maximum response of the controller. The maximum response is defined as the maximum rate at which a steady state frequency error can be corrected. With an effective speed droop signal equal to 2 per cent of the permanent speed droop setting and a sampling duration of 10 seconds, it takes the controller a minimum of 500 seconds to raise the speed droop curve from its full-load to its no-load position. The speed droop curve will be raised as a ramp function. Thus the controller may be optimized (or can minimize the time and frequency errors) for per unit ramp load changes, occurring on the individual generator, as rapidly as 1,000 seconds. If the rate of load change exceeds this value, in a manner which creates a time error at the termination of the load change, damped frequency and time error oscillations will result.

Therefore, to optimize the controllers performance on the system, it is necessary to have sufficient generation under the regulation of the controller such that the load variations do not exceed the former limit.

REFERENCES

1. K. G. Coreless and A. S. Alfred. "An Experimental Power System Simulator", Institution of Electrical Engineers, Proceedings, Vol. 105, Part A, 1958, pp. 503-511.
2. J. E. Van Ness and W. C. Peterson. "The Use of Analogue Computers in Power System Studies", The Transactions of the A.I.E.E., Power Systems and Apparatus, Part III, Vol. 75, 1956, pp. 238-242.
3. G. G. Richardson and T. C. Wang. "Use of the A.C. Network Analyzer and Electronic Differential Analyzer in the Study of Load-Frequency Control", Transactions of the A.I.E.E., Power Systems and Apparatus, Part III, Vol. 80, 1961, pp. 1143-1148.
4. O. W. Hanson. "A Study of Tie-Line Interaction and Load-Frequency Control", Master of Science Thesis in Electrical Engineering, University of Saskatchewan, April 1963.
5. Fitzgerald and Kingsley. "Electric Machinery", pp. 223-237, McGraw - Hill Inc., New York, 1961.
6. S. B. Crary, "Power System Stability" pp. 4-56, Vol. 1, J. Wiley and Sons Inc., New York, 1945-47.
7. M. Riaz. "Analogue Computer Representations of Synchronous Generators in Voltage Regulation Studies", Transactions of the A.I.E.E., Part III, Vol. 75, 1956, pp. 1178-1184.
8. G. Shackshaft. "A General-Purpose Turbo-Alternator Models, The Institution of Electrical Engineers, Paper No. 4160P.
9. J. E. Van Ness. "Synchronous Machine Analogues for Use with the Network Analyzer." Transactions of the A.I.E.E., Part III, Vol. 73, 1954, pp. 1056.
10. G. R. Rich. "Hydraulic Transients" Dover Pub., New York, 1963
11. L. O. Long. "Outlook on the Relation Between Load-Frequency Control and Turbine Governors on Interconnected Power Systems", The Engineering Journal, December 1961, pp. 42-54.
12. Woodward Governor Company. "The Control of Prime Mover Speed", Bulletin 25031, Part IV.
13. Timoshenko and MacCullough. "Elements of Strength of Materials", pp. 52-54, D. Van Nostrand Inc., Princeton, New Jersey, 1949

14. J. B. Nuttal. "Civil Engineering 434, 1961 Notes."
15. J. Parmakin. "Water-Hammer Analysis", Prentice-Hall Inc., New York, 1955
16. J. G. Tarboux. "Introduction to Electric Power Systems", The Haddon Craftsman Inc., Scranton, Pennsylvania
17. C. Concordia and L. K. Kirchmayer. "Tie-Line Power and Frequency Control of Electric Power Systems", pp. 133-146, A.I.E.E., Power Systems and Apparatus, Part III-A, 1954
18. M. Lowschitz-Garik and R. T. Weil. "D.C. and A.C. Machines", D. Van Nostrand Inc., Toronto, 1952
19. G. Gaskell. "Internal Report, The History of the Load and Frequency Regulator", issued by Calgary Power Limited.
20. L. M. Hovey. "The Use of an Electric Analogy Computer for Determining Optimum Settings of Speed Governors for Hydro Generating Stations", The Engineering Journal, May 1961, pp. 76-79.

APPENDIX A

DEFINITION OF SYMBOLS

Section I-2 The symbols utilized in the method of system simulation

I_d, I_q	the direct and quadrature axes currents
V_d, V_q	the direct and quadrature axes voltages
w	the electrical frequency
θ	the generator's internal torque angle
θ_1, θ_2	the internal torque angles of generators #1 and #2
δ_{12}	the synchronizing torque angle between generators

Section II-1 The symbols utilized in the generator and voltage regulator models

a	the turns ratio
E_f	the internal generator voltage
E_q	the voltage behind the quadrature reactance
F_a	the magnetomotive force created by the current in phase A
F_{da}	the component of F_a along the direct axis
F_{qa}	the component of F_a along the quadrature axis
F_{ds}	the stator magnetomotive force resulting from the three phase currents along the direct axis
F_{qs}	the stator magnetomotive force resulting from the three phase currents along the quadrature axis
i_a, i_b, i_c	the instantaneous phase currents
I_a, I_b, I_c	the maximum phase currents

I_{Br}	the rotor base current
I_d, I_q, I_o	the direct, quadrature and zero sequence currents
I_{kd}, I_{kq}	the direct and quadrature axes currents in the damper windings
I_f	the field current
I_{fo}	the field current required to generate rated voltage on the air gap line
L_{aao}	the total constant term of the self of phase A
L_{aa}, L_{bb}, L_{cc}	the self inductances of the phases
L_{ab}, L_{bc}, L_{ac}	the mutual inductances between phases
$L_{aff}, L_{bff}, L_{cff}$	the mutual inductances between the rotor and stator phases
$L_{af} = L_{gd}$	the direct axis mutual inductance between the stator phases and rotor as seen by the latter
L_{fl}	the field leakage inductance
L_{gaa}	the self inductance of phase A due to the air gap flux
L_{go}	the constant term of L_{gaa}
L_{g2}	the amplitude of the second harmonic of the variable term in L_{gaa}
L_{gq}	the quadrature axis mutual inductance between the stator phases and the rotor as seen by the latter
L_{kd}, L_{kq}	the direct and quadrature axes damper winding leakage reactances
N_a	the effective turns ratio
P_{gd}, P_{gq}	the permeance coefficients along the direct and quadrature axes
P_L	the generator output power
R_a	the armature resistance
R_{kd}, R_{kq}	the direct and quadrature axes damper winding resistances

R_f	the field resistance
T_u	the electrical air gap torque
V_{act}	the generator terminal voltage
V_{Br}	the rotor base voltage
V_d, V_q, V_o	the direct, quadrature and zero sequence voltages
V_f	the field voltage
V_{ref}	the desired terminal generator voltage
V_t	the terminal generator voltage
ω_o	the rated generator frequency
$\omega = \omega$	the generator frequency
$\Delta \omega$	the deviation from rated frequency
X_{ad}, X_{aq}	the direct and quadrature axes mutual reactances
X_{adr}	the mutual reactance referred to the rotor
X_{ads}	the mutual reactance referred to the stator
X_{al}	the armature leakage reactance
X_{Bs}	the stator base impedance
X_{Br}	the rotor base impedance
X_d, X_q	the direct and quadrature axes reactances
X_{kd}, X_{kq}	the direct and quadrature axes damper winding reactances
ϕ_{gba}	the mutual flux in phase B, resulting from the flux in phase A
ϕ_{gda}, ϕ_{gqa}	the air gap fluxes, along the direct and quadrature axes, resulting from the flux in phase A
ϕ_{gd}, ϕ_{gq}	the direct and quadrature fluxes, resulting from the currents in the three phases
$\lambda_a, \lambda_b, \lambda_c$	the per phase flux linkages
$\lambda_d, \lambda_q, \lambda_o$	the direct, quadrature and zero sequence flux linkages

$\lambda_{dr}, \lambda_{qr}$	the direct and quadrature axis flux linkages referred to the rotor
$\lambda_{kd}, \lambda_{kq}$	the direct and quadrature axes damper winding flux linkages
δ	the synchronizing torque angle between machines
θ	the generator's internal torque angle
ϕ	the rotor position expressed in radians

Section II-2 The symbols utilized in the Speed Governing Model

B	the effective gain of the flyball assembly as viewed in terms of the electrical frequency
f_e	the electrical frequency
Δf_e	the deviation from the electrical frequency
f_{eo}	the rated electrical frequency
G	the turbine wicket gate position
H	the generator and turbine inertia constant
K	the gain of the governor
K'	the governor gain between the pilot valve and the servo motor valve
P_d	the sum of the self regulation of the turbine and load
P_l	the load power at rated frequency
P'_l	the load power at rated frequency plus the change in power resulting from the voltage regulator frequency relationship
$\frac{dP'_l}{df}$	the self regulation of the load
P_L	the load power
P_t	the turbine power at rated frequency
$\frac{dP_t}{df}$	the self regulation of the turbine

P_T	the turbine power
R.P.M.	the revolutions per minute
T_L	the load torque
T_T	the turbine torque
T_r	the dashpot relaxation time constant
T_{rr}	the rated machine torque
W_m	the mechanical frequency
ΔW_m	the deviation from rated mechanical frequency
W_{m0}	the rated mechanical frequency
WR^2/g	the moment of inertia
x_p	the pilot valve position
x_v	the servo motor position
δ_p	the effective permanent speed droop
δ'_p	the actual permanent speed droop
δ_t	the effective temporary speed droop
δ'_t	the actual temporary speed droop

Section II-3 The Symbols Utilized in the Hydro Model

a	the acceleration of the water in the conduit
A_p	the area of the conduit or pipe
ΔA_p	the change in area of the conduit or pipe
A_s	the area of the surge tank
A_r	the area of the reservoir
A_T	the area of the turbine
b	the velocity of sound in water
C	the equivalent electrical capacitance of the conduit

C_d	the discharge coefficient
C_s	the equivalent electrical capacitance of the surge tank
D_p	the diameter of the conduit or pipe
D_s	the diameter of the surge tank
E_l	the equivalent voltage to pressure head variations
E_s	the sending voltage
E_r	the receiving voltage
Δf	the electrical frequency deviation
f_f	the coefficient of pipe friction
F	the force on the water column
g	the force of gravity
G	the wicket gate position
ΔG	the change in wicket gate position that will deliver rated power to the turbine
H	the piezometric head
ΔH	the change in the piezometric head
H_o	the rated head
H_r	the piezometric head at the reservoir
ΔH_r	the change in the piezometric head at the reservoir
H_t	the piezometric head at the turbine
ΔH_t	the change in the piezometric head at the turbine
ΔH_{Ln}	the change in the piezometric head over a segment of the conduit
I	the current which is equivalent to the flow rate in the conduit
I_r	the receiving current

I_s	the sending current
K	the bulk modulus of elasticity
K_1	the approximate pipe friction coefficient
K_2	the approximate throttling loss coefficient
L	the length of the conduit
ΔL	the change in length of the conduit
L_e	the effective length of the conduit as seen by the turbine
L'	the inductance which is equivalent to the inertia of the water in the conduit
L_1	the length of the conduit from the turbine to the surge tank
L'_1	the inductance equivalent to the inertia of the water from the turbine to the surge tank
L_2	the length of the conduit from the surge tank to the reservoir
L'_2	the inductance equivalent to the water inertia from the surge tank to the reservoir
m	the line propagation constant
P/γ	the pressure energy
P_t	the turbine power at rated frequency
ΔP_t	the change in turbine power at rated frequency
Q	the flow rate
ΔQ	the change in flow rate
Q_o	the flow rate that yields rated turbine power
Q_p	the flow rate in the conduit
ΔQ_p	the change in flow rate in the conduit
Q_{po}	the flow rate in the conduit prior to a disturbance
Q_r	the flow rate at the reservoir
ΔQ_r	the change in flow rate at the reservoir

Q_s	the flow rate in the surge tank
Q_t	the flow rate in the turbine
ΔQ_t	the change in flow rate in the turbine
ΔQ_{t3}	the third harmonic component of the flow rate at the turbine
ΔQ_{Ln}	the change in flow rate over a segment of the conduit
R	the radius of the conduit
R_f	the resistance representing the pipe friction losses
R_t	the resistance representing the throttling losses
s	the Laplace operator
t'	the thickness of the conduit
T	the hoop stress on the conduit
ΔT	the change in hoop stress on the conduit
T_w	the water starting time
T'_w	the water starting time from the turbine to the surge tank
U	the water velocity
U_o	the water velocity which will yield rated turbine power
U_p	the water velocity in the conduit
U_{po}	the water velocity in the conduit prior to the application of a disturbance
U_t	the water velocity in the turbine
U_s	the water velocity in the surge tank
V_p	the volume of the conduit
V_c	the volume of the water
W	the electrical frequency
W_c	the water compressibility

X	the distance from the turbine toward the reservoir
Y	the shunt admittance
Z	the potential energy per unit weight
Z_0	the characteristic impedance of the line
Z_π	the series impedance
$Z_{\pi 3}$	the series impedance of the line to third harmonic variations
Z'_π	the shunt impedance of 1/2 the line
$Z'_{\pi 3}$	the shunt impedance of 1/2 the line to third harmonic variations
η	the efficiency
γ	the specific weight of water
ξ	the natural frequency of the water hammer oscillations
ρ	the density of water

Section II-4 The symbols utilized in the Load Model

(Note: Barred symbols represent vector quantities)

E_q	the voltage behind the quadrature reactance
I_d, I_q	the direct and quadrature axis currents
I_f	the field current
I_{kd}	the direct axis current component in the damper windings
I_{11}	the current flowing in generator #1 when E_{q1} is acting alone
I_{12}	the current flowing in generator #2 when E_{q1} is acting alone
I_{12d2}, I_{12q2}	the current generated by unit #1 along the direct and quadrature axes of unit #2, when E_{q1} is acting alone
I_{22}	the current flowing in generator #2 when E_{q2} is acting alone

I_{21}	the current flowing in generator #1 when E_{q2} is acting alone
z_{1ext}, z_{2ext}	the impedances between the generator terminals and the load
$\theta_{11}, \theta_{22}, \theta_{12}$	the angles associated with the impedances $z_{11}, z_{22},$ and z_{12}
ϕ_1, ϕ_2	the machine's datum angle or angular position
δ	the synchronizing torque angle between machines

Section II-5 The symbols utilized in the Controller Model

B	the effective gain of the flyball assembly in terms of the electrical frequency
C	the spring constant of the flyball assembly
$\sum k$	the summation of the effective speed droop signals
$\sum k'$	the summation of the effective speed droop signals applied to the flyball assembly
f	the electrical frequency
Δf	the deviation from the electrical frequency
f_o	the rated electrical frequency
$\Delta f_1, \Delta f_2$	the deviation from the no-load to the load frequency when the units are carrying loads M_1 and M_2
F'	the force applied to the dashpot of the governor by the dashpot frequency signal
m	the sampling duration
M	the system load
M_1	the load carried by unit #1
M_2	the load carried by unit #2
P_t	the turbine power at rated frequency
P_1	the load power at rated frequency

q	the movement of the governor dashpot in response to the equivalent dashpot frequency signal
s_{f1}, s_{f2}	the rate of change of frequency, as related to the load, on generators #1 and #2 respectively
s_{t1}, s_{t2}	the rate of change of the time droop, as related to the load, on generators #1 and #2 respectively
ΔT	the time error
$\Delta T'$	the activating time error as seen by the controller
V	the effective dashpot frequency signal
x_v	the servo motor position
δ'_t	the temporary speed droop

Section III-1 The symbols utilized in the Analysis of Results

B_L	the magnitude of the backlash signal in seconds
E_1, E_2	the voltage behind the synchronous reactance of generators #1 and #2 respectively, assuming a round rotor machine
f_o	the rated system frequency
Δf	the frequency error on the system
f_1, f_2	the functional relationship between the turbine output power of generators #1 and #2 respectively, the synchronizing torque and frequency datum angles
F_1, F_2	the functional relationship between the turbine output power of generators #1 and #2 respectively and the synchronizing torque angle
G_1, G_2	the functional relationship of the turbine output power of generators #1 and #2 respectively and the frequency datum angle
H	the generator and turbine inertia constant

H_1, H_2	the generator and turbine inertia constants of units #1 and #2 respectively
I_1, I_2	the output currents of units #1 and #2 respectively
I_L	the current absorbed by the load
k	the magnitude of the effective speed droop signals
K	the frequency error equal to the permanent speed droop
m	the sampling duration
ΔM_T	the system load change
ΔM_n	the load change absorbed by the n^{th} generator
M_{nr}	the rated capacity of the n^{th} generator
N	the rate of change of system frequency caused by the load change
P_d	the system damping coefficient
P_L	the power absorbed by the load simulated on the computer model
ΔP_T	the turbine output power
$\Delta P_{T1}, \Delta P_{T2}$	the turbine output power on units #1 and #2 respectively
R	the system load resistance
S_{fn}	the rated change of frequency, as related to load, on the n^{th} generator
S_{ln}	the controller response of the n^{th} unit
s	the laplace operator
t	the time in seconds
t_o	the minimum time required for the controller to raise the permanent speed droop curve from its full-load to its no-load position
t_1	the time interval from the start of the load change to until the controller corrects the time error

ΔT	the time error
ΔT_d	the time droop bias created by the load sharing feature
$\Delta T'$	the activating time error signal ($\Delta T - \Delta T_d$)
U	the equivalent damping signal created by the time droop bias
V	the dashpot frequency signal
v'	the damping signal ($V + u$)
w	the system frequency
W_a	the natural frequency of the synchronizing torque loop
y	the per unit load carried by the individual generator
X	$X_d + X_T + X_L$
X_d	the synchronous reactance of the generators
X_T	the transformer reactance
X_L	the line reactance
ζ	the synchronizing torque angle
δ_p	the permanent speed droop
ϕ_o	the frequency datum angle
ϕ_1, ϕ_2	the angles of generators #1 and #2 respectively, relative to ground
θ	the angle associated with the mutual impedance between the generators

APPENDIX B

VALUE OF THE MODEL PARAMETERS

Section II-1 Generator and Voltage Regulator Parameters

X_{ad}	1.1 per unit	Base MVA = 40.0
X_{ad} saturated	0.66 per unit	Base KV = 138.0
X_{al}	0.15 per unit	
X_{fld}	0.15 per unit	
X_{KLD}	0.04 per unit	
R_{fd}	0.0017 per unit	
R_{KD}	0.05 per unit	
R_{KQ}	0.05 per unit	
T'_d	1.5 seconds	
H	1.5 KW-seconds/KVA	

Machine Rating	40.0 MVA
K	1.00 per unit
K_2	5.00 per unit
K_3	0.2 per unit
T	0.1 seconds

Section II-2 Governor Parameters

K	5.00 per unit
ζ_T	0.214 per unit
T_r	3.00 seconds
P_d	3.92 per unit
ζ_p	Variable

Section II-3 Hydro-System Parameters

T_w Variable

Section II-4 Load Parameters

Z_L 2.0 $\angle 36^\circ$ per unit for light load
 1.0 $\angle 1^\circ$ per unit for heavy load

X_T 0.10 per unit

X_L 104 ohm's or 0.218 per unit for 150 miles of line. It is assumed that there are 75 miles of line, operating @ 138 KV, between each generator and the load. Further it is assumed that the line is capacitively compensated $X = 8/3 R$

R 38.9 ohm's or 0.084 per unit for 150 miles of line

Section II-5 Controller Model Parameters

Quadrant Time Constant 165 seconds

Backlash 1/50 seconds

Dead Zone Variable

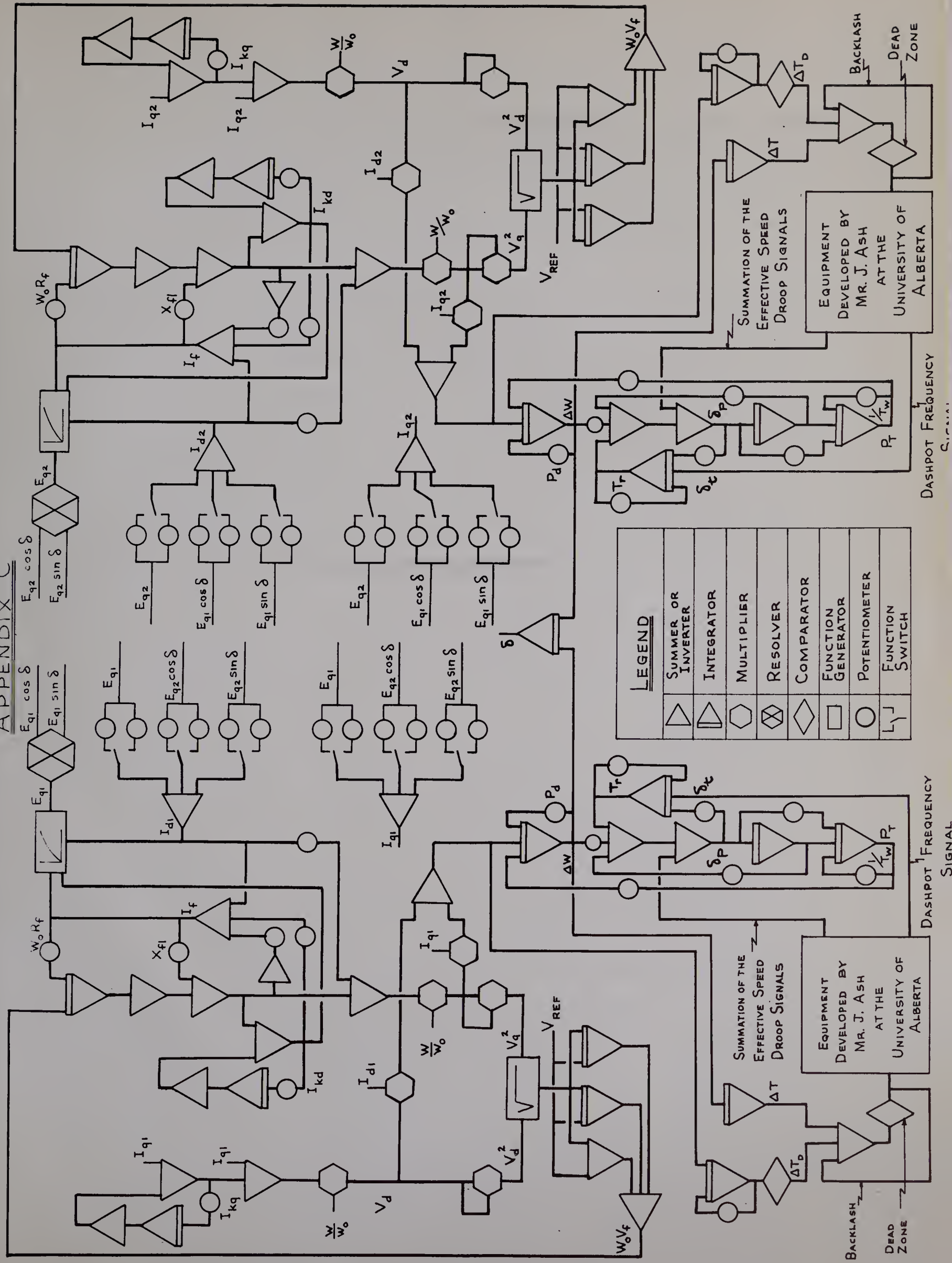
Effective Speed Droop Signal "

Dashpot Frequency Signal "

Percentage Load Setting "

Sampling Duration "

APPENDIX C



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